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EXTENSIONS OF MAPS IN SPACES WITH PERIODIC HOMEOMORPHISMS

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ABSTRACT. This is an announcement of some results on extension and retraction properties in the equivariant category of compact metric spaces with periodic maps of a prime period. If X and Y are spaces in this category, A is an equivariant closed subspace of X and $f: A \to Y$ is an equivariant map then the existence of an extension of f does not, in general, imply the existence of an equivariant extension. In the case, however, when A contains the fixed point set of the periodic map and $\dim (X - A) < \infty$, a condition for the existence of an extension is also sufficient for the existence of an equivariant extension. In particular, if follows that a finite-dimensional space Y in this category is an equivariant AR (resp. equivariant ANR) iff both Y and the fixed point set of the periodic map are AR's (resp. ANR's).

1. **Introduction.** Let us consider spaces with operators from Z_p, the cyclic group of order p; that is to say, pairs (X, a), where X is a space and $a: X \to X$ is a periodic homeomorphism of period p, $a^p = 1$. Such objects form a category which we denote by $\mathscr{A}_p$; a morphism $(X, a) \to (Y, b)$ in $\mathscr{A}_p$ is a map $f: X \to Y$ which is consistent with the homeomorphisms $a: X \to X$ and $b: Y \to Y$; it is also called an equivariant map. If (X, a) is an object of $\mathscr{A}_p$ and $Z \subset X$ is such that $aZ = Z$ then the periodic map a defines a periodic map $Z \to Z$ which sometimes will be denoted by a_Z; and (Z, a_Z) will be called an equivariant subspace of (X, a).

Some general extension theorems for spaces with operators exist in the literature, such as Gleason [3] and Palais [10]. Such theorems are generally some forms of the Tietze Extension Theorem carried over to an equivariant category. That is, they state that certain objects Y are injective, i.e. have the property that given any object X (suitably restricted) an equivariant closed subobject $A \subset X$ and an equivariant map $f: A \to Y$, there exists an equivariant extension $g: X \to Y$ of f over X. Such a space Y may be also called an absolute extensor in the category in question or an "equivariant absolute extensor" (EAE). Similarly, one can use the concepts of "equivariant absolute neighborhood extensor" (EANE), "equivariant absolute retract" (EAR) and "equivariant absolute neighborhood retract" (EANR) in a given equivariant category. The given group G of operators is usually assumed to be an orthogonal or a linear group and Y is assumed to be an equivariant convex subset of a vector space on which G acts linearly. Thus, for instance, the Tietze-Gleason-Palais Theorem asserts

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