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CONSTRUCTING ISOTOPIES IN NONCOMPACT 3-MANIFOLDS[1]

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Introduction. Let M be a noncompact, orientable 3-manifold with a (possibly empty) boundary ∂M. Suppose g and h are homeomorphisms of M onto itself. When is g isotopic to h? This question was essentially answered in the compact case by Waldhausen in [**3**]; roughly the answer given was—when g is homotopic to h. We will show that essentially the same answer can be given for a large and interesting class of noncompact manifolds; these manifolds include Whitehead-type contractible open subsets of R^3. Full proofs of the theorems stated below will be given elsewhere.

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Preliminaries. The ambient manifolds considered here are orientable, triangulable and 3-dimensional. By a *surface in M*, we mean a 2-dimensional, triangulable manifold which is properly imbedded in M. (Everything is considered from the piecewise linear point of view.) M is an *irreducible manifold* if every 2-sphere in M bounds a ball in M. For noncompact manifolds this implies that M is aspherical. A surface F in M or ∂M different from a 2-sphere is *incompressible* in M if $\pi_1(F) \to \pi_1(M)$ is a monomorphism. M is *boundary-irreducible* if each component of ∂M is an incompressible surface. Finally we need the notion of a *hierarchy* for a manifold. The triple $(F_j, U(F_j), M_j), j = 1, 2, \ldots,$ is a hierarchy for $M = M_1$ if each F_j is a compact incompressible orientable surface in M_j, $M_{j+1} = \mathrm{cl}(M_j - U(F_j))$, where $U(F_j)$ is a regular neighborhood of F_j in M_j [**4**], and $M - \bigcup_j \mathring{U}(F_j)$ is a collection of balls. If M is compact we require the sequence F_j to be finite. For M compact these surfaces have been constructed by Haken when M is irreducible and has an incompressible surface. Waldhausen uses the hierarchy to prove the isotopy theorem in the compact case.

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