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A GENERALIZATION OF THE HELLY SELECTION THEOREM

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1. **Introduction.** In this paper we consider sequences $\{y_k\}$ of real valued functions defined on an interval I. We are interested in finding conditions which when satisfied by the sequence $\{y_k\}$ guarantee the existence of a subsequence of $\{y_k\}$ which converges pointwise on I. With this in mind we make the following definition.

DEFINITION 1.1. Let $f:I \to R$ and consider the set $\mathscr{P}$ of all finite nonempty partitions $P = \{x_1, x_2, \ldots, x_n\}$ of I where $n \geqq 1$ and $x_1 < x_2 < \cdots < x_n$. We denote by $T(f)$ the oscillation of f on I and define it by

$$T(f) = \sup_{P \in \mathscr{P}} \left\{ \sum_{i=1}^{n} |f(x_i)| : (-1)^i f(x_i) > 0 \ \forall i \right.$$

$$\left. \text{or } (-1)^i f(x_i) < 0 \ \forall i \text{ or } (-1)^i f(x_i) = 0 \ \forall i \right\}.$$

For a function f which is nonnegative on I the oscillation of f on I, $T(f)$, is the supremum of f on I. It is not the case, however, that the set of f for which $T(f)$ is finite forms a Banach space with norm $T(f)$ since closure under addition is not satisfied. It is also not the case that the set of f for which $T(f)$ is finite forms a metric space with metric given by $d(f, g) = T(f - g)$ because the triangle inequality is not satisfied.

Our main result, for which we give a number of applications later, is the following.

THEOREM 1.2. *Let $\{y_k\}$ be such that $y_k : I \to R$. If $T(y_k - y_j) \leqq M$ for all k, j then $\{y_k\}$ contains a subsequence which converges pointwise on I.*

The original motivation for this theorem comes from the study of boundary value problems. In [3] the author showed, among other things, that if $\{y_k\}$ is a uniformly bounded sequence of continuous real valued functions defined on an interval I having the property that there exists a positive integer N such that y_k and y_j are not equal at more than N values of x for $k \neq j$ then y_k contains a subsequence which converges at every point in I. This result is a corollary of Theorem 1.2. A more complete description of the connection between such convergence theorems and the

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