

## RESEARCH ANNOUNCEMENTS

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### THE FIRST BETTI NUMBERS OF CERTAIN LOCALLY TRIVIAL FIBRE SPACES

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It is well known that, for a compact, oriented, homogenous coset space  $G/H$  arising from a compact, semisimple Lie group  $G$ , one has

$$b_1(G/H) \leq b_1(G) = 0.$$

In this note, we announce the following generalization of that result:

**THEOREM 1.** *Let  $\pi : E \rightarrow B$  be a locally trivial Riemannian fibre space,  $E$  and  $B$  compact, oriented Riemannian manifolds, with the fibres  $F = \pi^{-1}(b)$  immersed in  $E$  as minimal submanifolds. Then*

$$b_1(B) \leq b_1(E).$$

We outline the proof.

From Hodge-deRham theory, we have  $H^p(M, \mathbf{R}) \cong \mathcal{H}^p(M)$ , the space of harmonic  $p$ -forms on the compact, oriented Riemannian manifold  $M$ . In [4], we show

**THEOREM 2.** *Fix  $p \geq 1$ . If  $\varphi : E \rightarrow B$  is a locally trivial fibre space mapping between compact, orientable Riemannian manifolds satisfying  $\varphi^*\delta = \delta\varphi^*$  on all  $p$ -forms of the base manifold, then*

$$b_p(B) \leq b_p(E).$$

We also show

**THEOREM 3.** *Fix  $p \geq 1$ . Then  $\varphi : E \rightarrow B$ , a  $C^3$  map between arbitrary compact oriented Riemannian manifolds, commutes with the codifferential*

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