

REPRESENTATIONS OF ALGEBRAS BY CONTINUOUS SECTIONS

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ABSTRACT. This survey is concerned with the representation of discrete rings and topological algebras (such as C^* -algebras) by rings of continuous sections in sheaves of rings or fields of topological algebras.

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