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THE INNER PRODUCT OF PATH SPACE MEASURES CORRESPONDING TO RANDOM PROCESSES WITH INDEPENDENT INCREMENTS

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Let $X_1(t)$ and $X_2(t)$ be any two stochastically continuous, homogeneous random processes on $[0, T]$ with independent increments. It follows that $E(\exp(irX_k(t))) = \exp(tD_k(r))$, where

$$D_k(r) = i\alpha_k r - \beta_k \frac{r^2}{2} + \int_R \left(e^{iru} - 1 - \frac{iru}{1 + u^2} \right) d\sigma_k(u) \tag{1}$$

for some $\alpha_k \in R$, $\beta_k \geqq 0$, and Borel measure σ_k with $\int_R (u^2/(1 + u^2))\, d\sigma_k(u) < \infty$ (and with $\sigma_k(\{0\}) = 0$). We denote by ρ_k (resp. ρ_k^{*t}) the probability measure on R with characteristic function, $\exp(D_k(r))$ (resp. $\exp(tD_k(r))$), and by $\tilde{\rho}_k$ the probability measure on path space corresponding to X_k. $\tilde{\rho}_k$ is a Borel measure (with respect to the Skorokhod topology) on $D = D[0, T]$, the space of real valued functions on $[0, T]$ which are right-continuous and have left-hand limits, and may be defined in terms of ρ_k in the usual way.

If μ_1 and μ_2 are two measures on R (or D), we define $\sqrt{\mu_1}\sqrt{\mu_2}$ as the unique measure satisfying

$$\frac{d(\sqrt{\mu_1}\sqrt{\mu_2})}{dv} = \sqrt{\frac{d\mu_1}{dv}}\sqrt{\frac{d\mu_2}{dv}}$$

for any $v \gg \mu_1, \mu_2$; $(\sqrt{\mu_1} - \sqrt{\mu_2})^2$ thus denotes the (positive) measure, $(\mu_1 + \mu_2) - 2\sqrt{\mu_1}\sqrt{\mu_2}$. Given ρ_1 and ρ_2 as above, we define $N = N(\rho_1, \rho_2) = \int_R d(\sqrt{\sigma_1} - \sqrt{\sigma_2})^2$; N may be finite or infinite. If $N < \infty$, it is easily shown that $\int_R (|u|/(1 + u^2))\, d|\sigma_1 - \sigma_2| < \infty$ and we then define

$$\gamma = \gamma(\rho_1, \rho_2) = \frac{1}{2}\left(\alpha_1 - \alpha_2 - \int_R \frac{u}{1 + u^2}\, d(\sigma_1 - \sigma_2) \right).$$

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