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SURGERY ON POINCARÉ AND NORMAL SPACES[1]

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1. The object of this note is to announce a theory of surgery on Poincaré and normal spaces, with applications to Poincaré geometry and manifolds.

The present point of view on Poincaré surgery was outlined by W. Browder in the spring of 1969, and is purely homotopy-theoretic. The main missing ingredient in the program was Lemma 1.5, and the (considerable) machinery required for its application. Other approaches to Poincaré geometry have been made by N. Levitt [3], [4] using engulfing and manifold surgery, and by L. Jones [2] using patch structures (see 2.4 below).

Results in this area have also been obtained recently by W. Browder, A. S. Mischenko, and C. Morlet.

By a *Poincaré* space we mean one which satisfies duality with local coefficients (Wall [8]). Duality then gives a chain equivalence of the chains and cochains, which has a Whitehead torsion. This is the *torsion* of the space. The surgery groups L^s, L^h, etc. are encountered according to restrictions put on the torsion of the spaces considered. We neglect further mention of the torsion.

If X is Poincaré, one constructs the Spivak normal fibration ν_X^k with Thom class U, which is distinguished by a map $\rho : S^{n+k} \to T\nu_X$ such that $\rho_*[S^{n+k}] \cap U = [X]$. An abstraction is useful.

1.1. DEFINITION. A *normal* space is an X with a S^{k-1} fibration ξ_X and a map $\rho_X : S^{n+k} \to T\xi_X$. A *normal map* of normal spaces is a map $f : X \to Y$ with $\xi_X \simeq f^*\xi_Y$, and $\rho_Y = Tf^* \circ \rho_X$.

The *fundamental class* of X is $\rho_X[S^{n+k}] \cap U$, and has dimension n. A normal map is automatically of degree 1. Normal pairs (X, Y) have a map $\rho : S^{n+k} \to T\xi/T(\xi|Y)$.

The usefulness of normal spaces is twofold. First transversality holds for spherical fibrations in the normal category. Thus if pairs are used to define the normal bordism groups $\Omega_*^N(X)$, we get $\Omega_*^N(X) \simeq H_*(X; MSG)$. Secondly the mapping cylinder of a normal map is a normal pair. Therefore knowledge of the obstructions to improving a normal space to be Poincaré gives, by the first remark obstructions to Poincaré transversality, and Steenrod

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