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FOURIER INVERSION FOR SEMISIMPLE LIE GROUPS OF REAL RANK ONE[1]

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1. **Introduction.** Let G be a connected semisimple Lie group with finite center and let K be a maximal compact subgroup of G. We assume that $\operatorname{rank}(G) = \operatorname{rank}(K)$ and that $\operatorname{rank}(G/K) = 1$. Let T be a Cartan subgroup of G contained in K and let A be the noncompact Cartan subgroup of G constructed in [3f), §24]. Then (T, A) is a complete set of nonconjugate Cartan subgroups of G. We set $T' = T \cap G'$, $A' = A \cap G'$, where G' denotes the set of regular elements in G. We write $\mathfrak{g}$ and $\mathfrak{t}$ for the Lie algebras of G and T, and $\mathfrak{g}_{\boldsymbol{C}}$, $\mathfrak{t}_{\boldsymbol{C}}$ for the complexifications of $\mathfrak{g}$ and $\mathfrak{t}$. If $G_{\boldsymbol{C}}$ is the simply connected complex analytic group corresponding to $\mathfrak{g}_{\boldsymbol{C}}$, we assume that G is the real analytic subgroup of $G_{\boldsymbol{C}}$ corresponding to $\mathfrak{g}$.

For $f \in C_c^\infty(G)$, the invariant integral of f relative to the pair (G, T) is given by

$$\Phi_f^T(t) = \Delta_T(t) \int_{G/T} f({}^{\dot{x}}t)\, d_{G/T}(\dot{x}), \qquad t \in T',$$

where ${}^{\dot{x}}t = xtx^{-1}$, $x \in G$, and $d_{G/T}(\dot{x})$ is a suitably normalized invariant measure on G/T (see [3d)] and [**5**, vol. II, Chapter VIII]).

For fixed $t \in T'$, the map $f \mapsto \Phi_f^T(t)$, $f \in C_c^\infty(G)$, defines an invariant distribution Λ_t on G. In this paper, we give an explicit formula for the Fourier transform of Λ_t, that is, we determine a linear functional $\hat{\Lambda}_t$ such that

$$\Lambda_t(f) = \hat{\Lambda}_t(\hat{f}), \qquad f \in C_c^\infty(G).$$

In this context, we regard $\hat{f}$ as being defined on the space of invariant eigendistributions on G (see [3b)]). This formula is known in the case $G = \boldsymbol{SL}(2, \boldsymbol{R})$ ([**1**], [**2**], [3b)]). A similar inversion formula is given in [**4**] for the group $\boldsymbol{SL}(2, k)$, k a non-archimedean local field. We note that the proof of the formula in the latter case is quite different from that in the real case due to the form of the characters of the discrete series.

The Plancherel formula for G can be obtained directly from the inversion formula for $\Phi_f^T(t)$ by applying the differential operator $\Pi^T = \partial(\tilde{\omega})$ where $\tilde{\omega}$ denotes the product of the positive roots of the pair $(\mathfrak{g}_{\boldsymbol{C}}, \mathfrak{t}_{\boldsymbol{C}})$, and then using the fact that there exists a nonzero constant c independent of f such

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