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ON THE NONSEPARABLE THEORY OF BOREL AND SOUSLIN SETS

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Introduction. All spaces considered are assumed to be metrizable. The terminology follows that of [**5**]. The letter k will denote an infinite cardinal.

The purpose of this note is to introduce the notions of k-Souslin and k-Borel sets and to announce certain basic results obtained in their study. The results reinforce the feeling that these are the "natural" Borel and Souslin sets to study in nonseparable spaces of weight k. The $\aleph_0$-Souslin and $\aleph_0$-Borel sets are the standard Souslin and Borel sets studied in separable spaces.

The k-Borel sets of a space of weight $\leqq k$ can be resolved into "hyper-Borel" classes which form an increasing transfinite sequence of type $\omega(k + 1)$, the first ordinal of cardinal $k + 1$. Moreover, there exist spaces for which these classes are strictly increasing. One important property of k-Borel sets which is not a property of Borel sets (in nonseparable spaces) is that a locally k-Borel subset of a space of weight $\leqq k$ is k-Borel.

Every k-Borel subset of a space is a k-Souslin subset. On the other hand, the complement of a k-Souslin subset may not be k-Souslin, so k-Souslin subsets need not be k-Borel. However, a general form of Souslin's theorem holds: if a set and its complement are k-Souslin in a complete space of weight $\leqq k$, then both are k-Borel.

Both the k-Borel and k-Souslin sets are shown to be related to the Baire space of weight k in a way analogous to the relationship which exists between the classical Souslin and Borel sets and the space of irrational numbers.

Proofs and details will appear in [**2**].

1. **σ-discrete bases and co-σ-discrete mappings.** We recall that a family $\mathscr{B}$ of subsets of a space X is *σ-discrete* if $\mathscr{B} = \bigcup_{n=1}^{\infty} \mathscr{B}_n$ where each $\mathscr{B}_n$ is (relatively) discrete in the sense that for each $x \in B \in \mathscr{B}_n$ there exists an open set O_X in X such that $O_X \cap B' = \varnothing$ whenever $B \neq B' \in \mathscr{B}_n$. (See [**8**, Lemma 1] for equivalent statements.) A collection $\mathscr{E}$ of subsets of X is said to possess a *σ-discrete base* if there exists a σ-discrete family $\mathscr{B}$ of (not necessarily open) subsets of X such that each set in $\mathscr{E}$ is a union of sets from $\mathscr{B}$.

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