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ON THE SPECTRUM OF ALGEBRAIC K-THEORY

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ABSTRACT. The groups $K_i(A)$ of Bass for $i < 0$ are identified as homotopy groups of the spectrum of algebraic K-theory. The spectrum itself is identified. Applications to Laurent polynomials and to K-theory exact sequences are given.

Quillen has recently proposed a K-theory for unital rings [**12**], [**13**]. He associates to a ring A a space $BG1(A)^+$ whose homology is that of the group $G1(A)$ and whose homotopy groups $\pi_i\, BG1(A)^+$ he defines as $K_i(A)$, $i \geqq 1$. The space $BG1(A)^+$ is known to be an H-space, and indeed an infinite loop space.

Hence one is motivated to define $K_i(A)$, for $i \in Z$, as $\pi_i(E(A))$ where $E(A)$ is the associated Ω-spectrum. This note describes $E(A)$ and identifies the groups $K_i(A)$, $i < 0$. In fact, we show that the groups $K_i(A)$ are exactly the groups $L^{-i}K_0(A)$ discussed in Bass' book [**3**, p. 664] for $i < 0$.

Recall from the work of Karoubi and Villamayor [**10**] the cone CA and suspension SA of a ring A. An infinite matrix is called permutant if it is an infinite permutation matrix times a diagonal matrix of finite type. The diagonal matrix is of finite type if its diagonal entries are chosen from a finite subset of the ring. The ring CA is the ring generated by permutant matrices. The cone CA contains the two-sided ideal $\tilde{A} = \bigcup_n M_n(A)$ and the quotient ring is called the suspension of A. We can now state our main result.

THEOREM A. *The space* $\Omega\ (BG1(SA)^+)$ *has the homotopy type of* $K_0(A) \times BG1(A)^+$.

COROLLARY. *For all* $i \in Z$ *we have* $K_i(A) = K_{i+1}(SA)$.

Since Karoubi [**9**] has already identified $K_0(S^iA)$ with Bass' groups $K_{-i}(A)$, the Corollary above completes the identification of Bass' groups with the negative homotopy of the spectrum $E(A)$.

In proving Theorem A we must first analyze the cone construction.

THEOREM B. *The space* $BG1(CA)^+$ *is contractible.*

This result generalizes work of Karoubi and Villamayor [**11**] who show that $K_i(CA) = 0$ for $i \leqq 2$. To prove Theorem B we observe that it suffices

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