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THE RANGE OF m-DISSIPATIVE SETS

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Let X be a real Banach space and X^* its dual space. We shall give some sufficient conditions for an m-dissipative set A to have range R_A all of X or to be dense in X. The theorems which we shall prove are the following:

THEOREM 1. *If A is a coercive, m-dissipative set on X, then $\bar{R}_A = X$.*

THEOREM 2. *In addition to the assumptions of Theorem* 1, *suppose that there is a compact operator c on X and a strictly increasing right-continuous function λ such that*

$$\lambda(0) = 0 \quad \text{and} \quad \lambda(\|x_1 - x_2\|) \leqq \|y_1 - y_2 - (cx_1 - cx_2)\|$$

whenever $[x_1, y_1], [x_2, y_2] \in A$. Then $R_A = X$.

THEOREM 3. *Let X be a reflexive Banach space. If A is a coercive, demiclosed, m-dissipative set on X, then $R_A = X$.*

DEFINITION. A mapping J of X into 2^{X^*} is said to be the *duality mapping* if $Jx = \{w \in X^*; \|w\| = \|x\|, w(x) = \|x\|^2\}$ for all $x \in X$.

It is easy to see that for each $x \in X$, Jx is a nonempty, closed, convex, bounded subset of X^*. Thus, for any $z \in X, x \in X$, there is $y \in Jx$, such that $y(z) = \inf\{w(z) : w \in Jx\}$ and we use $\langle z, x\rangle$ to denote $y(z)$.

DEFINITION. A is said to be a *dissipative set* on X if A is a subset of $X \times X$ such that for $[x_1, y_1], [x_2, y_2]$ in A, $\langle y_1 - y_2, x_1 - x_2\rangle \leqq 0$.

T. Kato [5] showed that the above definition is equivalent to the following: for every pair $[x_1, y_1], [x_2, y_2]$ in A and $t \geqq 0$,

$$\|x_1 - x_2 - t(y_1 - y_2)\| \geqq \|x_1 - x_2\|.$$

Hence, if A is a dissipative set then $(1 - tA)^{-1}$ is a nonexpansive mapping on $R_{(1-tA)}$ into X for $t \geqq 0$. We will say that A is *m-dissipative* if $R_{(1-tA)} = X$ for all $t \geqq 0$. It is known that A is m-dissipative if and only if A is dissipative and $R_{(1-A)} = X$ (see S. Ôharu [6]).

DEFINITION. A dissipative set A is said to be *coercive* if $A^{-1}(B) = \{y \in X; Ay \cap B \neq \emptyset\}$ is bounded whenever B is a bounded subset of X.

DEFINITION. A is said to be *demiclosed* if A has the property that $x_n \rightharpoonup x_0$, $y_n \to y_0$, $[x_n, y_n] \in A$, for all $n = 1, 2, \ldots$, implies $[x_0, y_0] \in A$.

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