BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 1, January 1972

RELATIVE IMAGINARY QUADRATIC FIELDS OF LOW CLASS NUMBER

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Communicated by Oscar Goldman, June 22, 1971

ABSTRACT. Let K be a normal totally real algebraic number field. Then it is possible to effectively determine all totally imaginary quadratic extensions of K of class number 1. A partial result for class number 2 is obtained.

Let K be a totally real algebraic number field and let h be a positive integer. It is a simple consequence of the Brauer-Siegel theorem [**2**] that there exist only finitely many totally imaginary quadratic extensions L of K having class number h. In this note, we will announce some results connected with the following:

Problem. For fixed K, determine effectively all totally imaginary quadratic extensions of class number h.

When $K = \boldsymbol{Q}$, this problem is equivalent to the determination of an effective procedure for classifying all imaginary quadratic fields of class number h. Even in this special situation, little is known. In 1966, Stark [**4**] settled the case $h = 1$ by proving that there are precisely 9 imaginary quadratic fields of class number 1. In 1970, the author [**3**], Baker [**1**], and Stark [**5**] succeeded in giving an effective upper bound for the absolute value of the discriminant of an imaginary quadratic field of class number 2. At the present time, the solution to the problem, for $K = \boldsymbol{Q}$, $h \geqq 3$, is not known.

Recently, J. Sunley [**6**] has proved the following result:

THEOREM [S]. *Let K be a fixed totally real algebraic number field and let L be a totally imaginary quadratic extension of K, having class number h_L and discriminant d_L. There exists an effectively computable constant $c = c(K, h)$ such that if $h = h_L$, then*

$$|d_L| \leqq c,$$

with the possible exception of one field L.

Our main results concern the possible exceptional field L.

THEOREM 1. *Let all notations be as in* [S], *and assume that K is normal. Then the exceptional field L (if it exists) must be normal over $\boldsymbol{Q}$.*

AMS 1969 *subject classifications.* Primary 1065; Secondary 1068.

Key words and phrases. Number field, class number, imaginary quadratic extension.

[1] Research supported by NSF Grant GP-20538.