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THREE NEW EXAMPLES OF COMPACT MANIFOLDS ADMITTING RIEMANNIAN STRUCTURES OF POSITIVE CURVATURE

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1. **Introduction.** The purpose of this note is to announce several new results on compact homogeneous spaces that admit invariant Riemannian structures of strictly positive curvature (that is, all the sectional curvatures are bounded below by a positive constant). The main result that we announce is that the manifolds of flags in complex, quaternionic and Cayley three-space admit homogeneous Riemannian structures of strictly positive curvature. As coset spaces these manifolds are given as $SU(3)/T$ (T a maximal torus of $SU(3)$), $Sp(3)/SU(2) \times SU(2) \times SU(2)$ and $F_4/\mathrm{Spin}(8)$ (note that $F_4/\mathrm{Spin}(9)$ is the Cayley plane). The example $F_4/\mathrm{Spin}(8)$, which we have called the manifold of flags in Cayley three-space was not on our list in [**2**]. We take this chance to correct our error by including the pair $(F_4, \mathrm{Spin}(8))$ in the list of Theorem 5.1 of [**2**].

Each of the above manifolds have Euler characteristic 6 and since they respectively have dimension 6, 12 and 24 it is not hard to see that these spaces are not homeomorphic with rank one symmetric spaces. We also point out that the metrics of positive curvature on $SU(3)/T$ are Hermitian relative to a complex structure on $SU(3)/T$.

The proofs of these results and those announced in [**2**] will appear in [**3**].

2. **Condition** (III). Let G be a compact and connected Lie group and let K be a closed subgroup of G. Let $\mathfrak{g}$ be the Lie algebra of G and let $\mathfrak{k}$ be the Lie algebra of K. Let $(\ ,\ )$ be an $\mathrm{Ad}(G)$-invariant inner product on $\mathfrak{g}$ (Ad is the adjoint action of G on $\mathfrak{g}$). Let $\mathfrak{p}$ be the orthogonal complement to $\mathfrak{k}$ in $\mathfrak{g}$ relative to $(\ ,\ )$.

We say that (G, K) satisfies condition (III) if

(1) $\mathfrak{p} = V_1 + V_2 + V_3$ an orthogonal direct sum relative to $(\ ,\ )$ and V_i is $\mathrm{Ad}(K)$-invariant for $i = 1, 2, 3$.

(2) $[V_i, V_i] \subset \mathfrak{k}$ for $i = 1, 2, 3$.

(3) $[V_i, V_j] \subset V_k$ if $i \neq j$ and if $\{i, j, k\} = \{1, 2, 3\}$.

(4) If $x = x_1 + x_2 + x_3, y = y_1 + y_2 + y_3$ with x_i, y_i in V_i for $i = 1, 2, 3$ and if $[x, y] = 0$, $x \wedge y \neq 0$ then $[x_3, y_3] \neq 0$.

THEOREM 2.1. *If (G, K) satisfies condition* (III) *then G/K admits a G-invariant Riemannian structure of strictly positive curvature.*

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