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INCOMPLETE NORMED ALGEBRAS

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The Jacobson theory for rings is not well adapted to the study of a topological ring R because the basic items of that theory such as the primitive ideals, the modular maximal right ideals and the radical need not be closed in R. In special cases, such as Banach algebras [**7**] or more generally Q-rings [**3**], all these items are closed while in some other cases such as locally compact rings [**5**] one has the radical closed but not necessarily the primitive ideals or the modular maximal right ideals.

We approach the study of ideal theory for R by using only closed one-sided or two-sided ideals (ideals are two-sided unless otherwise specified). A number of approaches are outlined below each providing useful conclusions. The results obtained are sharpest in case R has the additional structure of being an (incomplete) normed algebra. Detailed proofs will appear elsewhere.

Examples show [**10**] that a right ideal in R can be a maximal-closed modular right ideal without being a maximal modular right ideal even for normed algebras. Call an ideal K *topologically primitive* if it has the form $K = (M:R) = \{x \in R : Rx \subset M\}$ and denote by the *topological radical*, top rad R, the intersection of all topologically primitive ideals. A theory of topologically primitive ideals is developed. In some ways it differs from the usual theory for primitive ideals. For example, if K is a topologically primitive ideal in R and I is an ideal in R, then $K \cap I$ need not be a topologically primitive ideal in I. Here we focus attention on the following question. Let $\mathfrak{P}_r(\mathfrak{P}_l)$ be the intersection of the maximal-closed modular right (left) ideals. Is $\mathfrak{P}_r = \mathfrak{P}_l = \text{top rad } R$ (in analogy with primitive ideal theory)?

We find the answer to be affirmative for certain classes of normed algebras. A first rather easy case is for a normed algebra which is a dense ideal in a Banach algebra.

For a complex normed algebra B with involution $x \to x^*$ let P denote the closure in the set of all selfadjoint elements of the set of all finite sums of elements of the form x^*x (P is the "positive cone" of B). In these terms we have the following result.

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