

RESEARCH ANNOUNCEMENTS

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ON THE TYPE OF ASSOCIATIVE H -SPACES

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Let X denote a connected, simply connected associative H -space of finite dimension and type which is not acyclic. By a well-known theorem of Hopf, $H_*(X; \mathbb{Q})$ is an exterior algebra on generators of odd degree, $E(x_1, x_2, \dots, x_k)$. We define the rank of X to be the number of generators, k , and the type of X to be the k -tuple $[n_1, n_2, \dots, n_k]$ where $n_i = \deg x_i$ and, for convenience, $n_i \leq n_{i+1}$.

There are two kinds of questions one may ask. First, given that the rank of X is some fixed small number, what possible types may occur? Results for rank $X \leq 4$ can be found in [2], [3], and [4]. Secondly, one may ask what possible types can occur if the greatest entry in the type of X is known. In particular, if the greatest entry is large are there associative H -spaces whose types continue the sequence of types of exceptional Lie groups as has been conjectured? The present paper gives some answers to both kinds of questions. The main results are the following:

THEOREM 1. *If rank $X \leq 5$ then either X has the type of a Lie group or has one of the types $[3, 5, 7, 11, 15]$, $[3, 7, 11, 11, 15]$ or $[3, 5, 5, 7, 9]$.*

THEOREM 2. *Assume that the type of X has no entry occurring more than once and that the greatest entry, $2n - 1$, is greater than 59.*

(i) *If n is odd then the type of X has all the numbers $\{2i - 1 | 2 \leq i \leq n\}$ among its entries.*

(ii) *If n is even and $n/2$ is not a power of 3 then the type of X has all numbers $\{4i - 1 | 1 \leq i \leq n/2\}$ among its entries.*

(iii) *If n is even and $n/2$ is a power of 3 then the type of X has all numbers $\{4i - 1 | 1 \leq i \leq (n/2) - 1\}$ among its entries.*

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