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INTEGRATION OF COMPLEX VECTOR FIELDS[1,2]

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1. **The problem.** First we describe the local problem. Let $L_1, \ldots, L_m$ be vector fields defined in a neighborhood U of the origin in $\boldsymbol{R}^n$ by the expressions:

$$L_k = \sum_{j=1}^{n} a_k^j \frac{\partial}{\partial x_j}, \qquad k = 1, \ldots, m, \tag{1.1}$$

where the a_k^j are infinitely differentiable complex-valued functions on U. The local problem is to "solve" the equations

$$L_k u = f_k, \qquad k = 1, \ldots, m. \tag{1.2}$$

That is, given functions $f_1, \ldots, f_m$ we wish to find conditions for the existence of a function u satisfying (1.2); further we wish to describe the set of functions satisfying (1.2) and also their dependence on the f_k, especially with respect to regularity properties. First consider the homogeneous case when $f_k = 0$, i.e.,

$$L_k u = 0, \qquad k = 1, \ldots, m. \tag{1.3}$$

Any function u satisfying all the above equations must also satisfy the equations

$$[L_k, L_h]u = L_k L_h u - L_h L_k u = 0. \tag{1.4}$$

Thus it is reasonable to assume that the space spanned by the vector fields $L_1, \ldots, L_m$ is closed under the bracket operation.

Condition A. This condition is satisfied if

$$[L_k, L_h] = \sum_j a_{kh}^j L_j \tag{1.5}$$

where the $a_{kh}^j \in C^\infty(U)$.

From (1.2) we obtain

$$[L_k, L_h]u = L_k f_h - L_h f_k$$

and hence (1.5) yields

$$L_k f_h - L_h f_k = \sum_j a_{kh}^j f_j, \qquad 1 \leqq k < h \leqq m. \tag{1.6}$$

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