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SOME FUNDAMENTAL THEOREMS IN COHOMOLOGY THEORY OF TOPOLOGICAL TRANSFORMATION GROUPS

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Ever since the beautiful fixed-point theorem of P. A. Smith for prime periodic transformations on homology spheres, algebraic topologists have been trying to extend his result in the following two directions:

(i) to generalize the transforming groups to general compact Lie groups, and

(ii) to generalize the spaces to those of more complicated cohomologies.

Unfortunately, the past trials are not very successful and the results so obtained are rather disappointing. However, it seems to me that the main reason for such failures is not due to the lack of powerful techniques nor the lack of proper setting, but rather the absence of a suitable viewpoint and a workable approach. As it was pointed out in [5], a perfectly natural approach is to follow the general ideas of I. Schur, É. Cartan and H. Weyl which worked so nicely for the study of linear transformations of compact Lie groups. However, in order to make such a simple minded idea of via "maximal tori" truly workable and penetrating, one needs some exceedingly strong and deep theorems for topological actions of tori so as to fill the missing cornerstone of the Schur lemma for topological transformation groups. The results of [2], [3], [4] clearly demonstrate that one may view the cohomology theory of topological transformation groups as a type of characteristic class theory for fibre bundles with topological G-spaces of a given cohomology type as fibre. And the central results that one needs to set up the geometric weight systems will be a certain type of *splitting principle* for *topological actions* of *tori.*

Following A. Borel, we shall denote the twisted product of a G-space X and the total space of universal G-bundle, E_G, by X_G. Namely, $X_G = X \times_G E_G$ is the total space of the universal bundle

$$X \to X_G \xrightarrow{\pi} B_G$$

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