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DISSIPATIVE PERIODIC PROCESSES

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1. **Introduction.** There has been recently the development of a general theory of dynamical systems going beyond ordinary differential equations which includes functional differential equations, partial differential equations, systems arising in the theory of elasticity, etc. A large number of examples of such dynamical systems and more complete references can be found in the paper [1] by Hale. For extensions to periodic systems and certain nonautonomous systems see [2] and [3]. Applications can be found in [4]–[7].

In this same spirit we develop here a general theory of dissipative periodic systems that applies to systems which "smooth" initial data (retarded functional differential equations, for example). This extends the work of Billotti in [8]. Nonlinear ordinary differentials which are periodic and dissipative were studied by Levinson in [9] in 1944, and more general results can be found in [10] and [11]. For ordinary differential equations one studies the iterates of a map T of a state space into itself where the map T is topological and the space is locally compact (n-dimensional Euclidean space). However, for the applications we have in mind the solutions will be unique only in the forward direction of time and the state spaces are not locally compact. Because of this the generalization of the results for ordinary differential equations is by no means trivial.

The basic theory of disspative periodic processes on Banach spaces is developed in §§2 and 3. How this applies to retarded functional differential equations of retarded type is discussed briefly in §4.

2. **Dissipative mappings.** Let R denote the real numbers, R^+ the nonnegative reals, and let X be a Banach space with norm $\|\cdot\|$.

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