BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

THE INTERPOLATORY BACKGROUND OF THE EULER-MACLAURIN QUADRATURE FORMULA

BY I. J. SCHOENBERG AND A. SHARMA

Communicated by Fred Brauer, May 14, 1971

In [4] the first named author discussed the explicit solutions of the cubic spline interpolation problems. We are now concerned with quintic spline functions. Let $S_5[0, n]$ denote the class of quintic spline functions $S(x)$ defined in the interval $[0, n]$ and having the points $0, 1, \cdots, n-1$ as knots. This means that the restriction of $S(x)$ to the interval $(\nu, \nu+1)$ $(\nu=0, \cdots, n-1)$ is a fifth degree polynomial, and that $S(x)\in C^4[0, n]$. With these functions we can solve uniquely the following three types of interpolation problems.

1. *Natural quintic spline interpolation.* We are required to find $S(x)\in S_5[0, n]$ such as to satisfy the conditions

$$S(\nu) = f(\nu) \qquad (\nu = 0, \cdots, n), \tag{1}$$

$$S'''(0) = S^{(4)}(0) = S'''(n) = S^{(4)}(n) = 0. \tag{2}$$

2. *Complete quintic spline interpolation.* We are to find $S(x) \in S_5[0, n]$ so as to satisfy the conditions

$$S(\nu) = f(\nu) \qquad (\nu = 0, \cdots, n), \tag{3}$$

$$S'(0) = f'(0), \quad S''(0) = f''(0), \quad S'(n) = f'(n), \quad S''(n) = f''(n). \tag{4}$$

3. *The interpolation of Euler-Maclaurin data.* Here we seek $S(x)\in S_5[0, n]$ such that

$$S(\nu) = f(\nu) \qquad (\nu = 0, \cdots, n), \tag{5}$$

$$S'(0) = f'(0), \quad S'''(0) = f'''(0), \quad S'(n) = f'(n), \quad S'''(n) = f'''(n). \tag{6}$$

In the present note we propose to do for quintic spline interpolation what was done in [4] for cubic interpolation. Also the method used is similar; in the present case we derive our results from the 5th degree case of cardinal spline interpolation discussed in [2]. We describe here the results concerning the third problem (5) and (6).

The foundation of our discussion is the quintic B-spline

AMS 1970 *subject classifications*. Primary 41A15, 65B15.