BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

LOCALIZATION AND COMPLETION IN HOMOTOPY THEORY[1]

BY A. K. BOUSFIELD AND D. M. KAN

Communicated by E. H. Brown, June 1, 1971

1. **Introduction.** For each *space* X (i.e. simplicial set with only one vertex) and *solid ring* R (i.e. commutative ring with 1, for which the multiplication map $R\otimes R\to R$ is an isomorphism [3]) we shall construct, in a *functorial* manner, a space $X\hat{_R}$, the *R-completion* of X, and discuss some of its properties. The proofs will be given elsewhere.

If $R\subset Q$ (i.e. R is a subring of the rationals) and $\pi_1X=0$, then $\pi_*X\hat{_R}\approx\pi_*X\otimes R$ and $X\hat{_R}$ is a *localization* in the sense of [7], [9] and [11].

If $R=Z_p$ (the integers modulo a prime p), $\pi_1X=0$ and π_nX is finitely generated for all n, then $\pi_*X\hat{_R}$ is the usual p-profinite completion of π_*X and $X\hat{_R}$ is a *p-completion* in the sense of [8] and [11].

This note is, in some sense, a continuation of [2]. However, our present construction is (although *homotopically equivalent to*) completely different from the one of [2] and has the advantage that it can easily be generalized to a *functorial* notion of *fibre-wise R-completion.* In [2] we used *cosimplicial* methods, while here the basic tool is that of

2. **The R-completion of a group.** To define this notion we call a group N an *R-nilpotent* group if N has a *central series*

$$1 = N_k \subset \cdots \subset N_j \subset \cdots \subset N_0 = N$$

such that for each j the quotient group N_j/N_{j+1} *admits an R-module structure.* The R-completion of a group G then is the group $G\hat{_R}$ obtained by combining Artin-Mazur [1, §3] with an inverse limit, i.e. by taking the inverse limit [1, p. 147] of the functor which assigns to every homomorphism $G\to N$, where N is R-nilpotent, the group N, and to every commutative triangle

$$\begin{array}{ccc} & & N \\ & \nearrow & \downarrow \\ G & & \\ & \searrow & \\ & & N' \end{array}$$

AMS 1969 *subject classifications.* Primary 5540.

Key words and phases. Relative R-completion of groups, R-completion of spaces, fibre-wise R-completion, R-nilpotent groups, R-nilpotent spaces.

[1] This research was supported by the National Science Foundation.