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ON THE EXISTENCE OF SPECIFIED CYCLES IN COMPLEMENTARY GRAPHS[1]

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It is well known that in any gathering of six people, there are three people who are mutual acquaintances or three people who are mutual strangers. This statement has the graph-theoretic formulation that for any graph G of order 6, either G or its complement $\overline{G}$ has a triangle. Furthermore, this statement is not true in general if "six" is replaced by a smaller integer.

The Ramsey number $r(m, n)$ may be considered a generalization of the above statement. For integers m, $n \geqq 2$, the number $r(m, n)$ is defined as the least integer p such that for any graph G of order p, either G contains the complete subgraph K_m of order m or $\overline{G}$ contains K_n. Hence, $r(3, 3) = 6$. It is a trivial observation that $r(m, n) = r(n, m)$, and $r(2, n) = n$ for all $n \geqq 2$. Despite the fact that a great deal of research has been done on Ramsey numbers, only six values $r(m, n)$ have been determined for m, $n \geqq 3$ (see [1]); namely, $r(m, n)$ is known (for m, $n \geqq 3$) only when $(m, n) = (3, 3)$, $(3, 4)$, $(3, 5)$, $(3, 6)$, $(3, 7)$, $(4, 4)$. Thus, no general formula for $r(m, n)$ has been determined for a fixed $m \geqq 3$ and arbitrary n; indeed, no such formula has even been conjectured.

There is a generalization of the problem of the three acquaintances and three strangers which is different from that which leads to the Ramsey numbers but which is just as natural. If we denote an n-cycle by C_n, then the above problem may be stated as: Given a graph G of order 6, either G or $\overline{G}$ contains C_3. This suggests the following generalization. For $m,n \geqq 3$, the number $c(m, n)$ is defined as the least integer p such that for any graph G of order p, either G contains C_m or $\overline{G}$ contains C_n. Of course, $c(3, 3) = 6$. We wish now to announce formulas for $c(3, n)$, $c(4, n)$, and $c(5, n)$ for all $n \geqq 3$.

THEOREM 1. *If $n \geqq 3$, then*

$$\begin{aligned} c(3, n) &= 6 && \text{if } n = 3, \\ &= 2n - 1 && \text{if } n \geqq 4. \end{aligned}$$

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