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AUTOMORPHISMS OF A FREE ASSOCIATIVE ALGEBRA OF RANK 2

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We announce here that the answer to the following conjecture [3, p. 197] is in the affirmative:

If R is a Generalized Euclidean Domain then every automorphism of the free associative algebra of rank 2 over R is tame, i.e. a product of elementary automorphisms.

We state here the necessary steps to prove the conjecture; detailed proofs will appear in [4] and [5].

Notation. R stands for a commutative domain with 1;

$R\langle x, y\rangle$ is the free associative algebra of rank 2 over R, on the *free* generators x and y;

$R(\bar{x}, \bar{y})$ is the polynomial algebra over R on the *commuting* indeterminates $\bar{x}$ and $\bar{y}$.

We write $R\langle x, y\rangle$ as a bigraded algebra

$$R\langle x, y\rangle = \bigoplus_{r \geqq s \geqq 0} \mathcal{A}_r^s$$

where the subindex denotes the homogeneous degree and the upper index stands for the degree in x. We will write $P = \sum P_r^s$ where $P_r^s \in \mathcal{A}_r^s$ for every $P \in R\langle x, y\rangle$.

The elementary automorphisms of $R\langle x, y\rangle$ are by definition the following:

(i) $\sigma \in \mathrm{Aut}_R(R\langle x, y\rangle)$; $\sigma(x) = y$; $\sigma(y) = x$.

(ii) $\varphi_{\alpha,\beta} \in \mathrm{Aut}_R(R\langle x, y\rangle)$, α, β units of R;

$$\varphi_{\alpha,\beta}(x) = \alpha x; \qquad \varphi_{\alpha,\beta}(y) = \beta y.$$

(iii) $\tau_{f(y)} \in \mathrm{Aut}_R(R\langle x, y\rangle)$, where $f(y)$ is any polynomial that does not depend on x;

$$\tau_{f(y)}(x) = x + f(y); \qquad \tau_{f(y)}(y) = y.$$

In a completely parallel way one defines the elementary automorphisms of $R(\bar{x}, \bar{y})$.

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