

PRIMITIVE SUBALGEBRAS OF EXCEPTIONAL LIE ALGEBRAS

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The object of this paper is to classify (up to inner automorphism) the primitive, maximal rank, reductive subalgebras of the (complex) exceptional Lie algebras. By primitive we mean that the subalgebras correspond to (possibly disconnected) maximal Lie subgroups. In [3], the corresponding classification for the (complex) classical Lie algebras was completed, as was the classification for the non-reductive, maximal rank, subalgebras of all the simple Lie algebras.

Using case by case techniques and some more general results given later, we prove the following theorem:

THEOREM 0. *The primitive, maximal rank, reductive subalgebras of the exceptional (complex, simple) Lie algebras are listed (up to conjugacy by an inner automorphism) in the table below. Further, all subalgebras isomorphic to one of these are conjugate by an inner automorphism.*

Algebra	Primitive subalgebras
E_8	$A_1 \oplus E_7, A_1^8, A_2 \oplus E_6, A_2^4, A_4^2, D_4^2, D_8, A_8, T^8$
E_7	$A_1 \oplus D_6, A_1^3 \oplus D_4, A_1^7, A_2 \oplus D_5, A_2^3 \oplus T^1, A_7; E_6 \oplus T^1, T^7$
E_6	$A_1 \oplus A_5, A_2^3; D_4 \oplus T^2, D_5 \oplus T^1, T^6$
F_4	$A_1 \oplus C_3, A_2^2, B_4, D_4$
G_2	A_1^2, A_2

T^k denotes the center of the subalgebra, where k is the dimension of that center. The other superscripts refer to the number of summands of the corresponding algebra.

We note that Theorem 5.5 (p. 148) in the reductive case of Dynkin [2] is incorrect. In particular $A_3 \oplus D_5$, $A_5 \oplus A_2 \oplus A_1$, $A_7 \oplus A_1$ in E_8 , $A_3^2 \oplus A_1$ in E_7 and $A_3 \oplus A_1$ in F_4 are not maximal subalgebras. (See [2, Table 12, p. 155].)

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We now present some basic notation and a characterization of