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ERGODIC THEORY AND THE GEODESIC FLOW ON SURFACES OF CONSTANT NEGATIVE CURVATURE

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1. **Introduction and abstract.** Famous investigations on the theory of surfaces of constant negative curvature have been carried out around the turn of the century by F. Klein and H. Poincaré in connection with complex function theory. The theory of the geodesics in the large on such surfaces was developed later in the famous memoirs by P. Koebe. This theory is purely topological. The measure-theoretical point of view became dominant in the late thirties after the advent of ergodic theory, and the papers of G. A. Hedlund and E. Hopf [2] on the ergodic character of the geodesic flow came into being. The present paper is an elaboration of the author's Gibbs lecture of this year and at the same time of the author's paper of 1939 on the subject, at least of its part concerning constant negative curvature. The author has had complaints about too much detail missing in the presentation of the material in the latter paper. This has been rectified in the present paper.

The very deep and very important recent papers by Sinai and Anosov on the subject are not touched upon in this paper. They are connected with the second phase of ergodic theory which came into being by the introduction of the notion of entropy into ergodic theory.

2. **Two-dimensional hyperbolic geometry.** The well-known Beltrami-Poincaré model of the hyperbolic plane is the interior of the unit circle endowed with the Riemannian metric

$$ds^2 = \frac{dx_1^2 + dx_2^2}{(1 - x_1^2 - x_2^2)^2} \tag{1}$$

which has curvature minus one. The isometries in this geometry are those Moebius transformations that map $x_1^2+x_2^2<1$ onto itself. An isometry is completely determined by the requirement that it carry a given orthogonal two-leg (ordered pair of directed line-elements with the same carrier point) again into such a thing. Line-elements

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