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STANDARD MODELS OF SET THEORY WITH PREDICATION

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1. **Introduction.** We consider another axiomatization of set theory. It is a first-order theory with equality, the membership relation, a new binary relation called *predication* and denoted by backwards epsilon, and a constant V denoting the universe of sets. Sets are defined to be elements of V. Classes are defined to be collections of sets. The variables P, Q, R are defined to range over classes. Thus, $\forall P\ \Phi(P)$ is short for

$$\forall x(\forall y(y \in x \rightarrow y \in V) \rightarrow \Phi(x)).$$

We only consider classes on the left of predication. The axioms are the universal closures of

(A) $x \in y \in V \rightarrow x \in V$,

(B) $\forall x \in V(P \ni x \leftrightarrow x \in P)$,

(C) $\forall x \in V(P \ni x \leftrightarrow Q \ni x) \rightarrow P = Q$,

(D) $\forall x_1, \cdots, x_n \in V\ \exists Q\, \forall y(Q \ni y \leftrightarrow \Phi(P_1, \cdots, P_n; x_1, \cdots, y_1))$,

where Φ is a formula such that

(i) all its free variables are displayed,

(ii) the P's are the only variables occurring on the left in predication,

(iii) the P's occur only on the left in predication, and

(iv) V does not occur.

We also add as axioms that

(E) V satisfies the axiom of choice and

(F) V satisfies the axiom of regularity.

This theory was first considered by the second author and will be presented in his thesis along with the proofs of the following theorems in the theory.

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