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BOUNDARY BEHAVIOR OF SOLUTIONS OF PARABOLIC EQUATIONS WITH DISCONTINUOUS COEFFICIENTS[1]

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Introduction and results. Consider the parabolic operator in divergence form

$$Lu = u_t - \{a_{ij}(x, t)u_{x_i}\}_{x_j} \tag{1}$$

where we have employed the convention of summation over repeated indices. Here $x=(x_1, x_2, \cdots, x_n)$ denotes a point in E^n with $n \geqq 1$ and t denotes a point on the real line. Assume that the coefficients of (1) are bounded measurable functions of (x, t) in $S=E^n \times (-1, 2T)$ for some $T>0$ and that there is a constant $\lambda>0$ such that $a_{ij}(x, t)z_iz_j \geqq \lambda |z|^2$ almost everywhere in S for all $z \in E^n$.

Let G be a bounded domain in $E^n \times (0, T)$. We prove that to each function $f \in C(\partial G)$ there corresponds a weak solution u of the boundary value problem

$$\begin{aligned} Lu &= 0, && \text{in } G, \\ u(x, t) &= f(x, t), && \text{on } \partial G. \end{aligned} \tag{2}$$

The precise definition of weak solution is given below; here it suffices to know that $u \in C(G)$. The notion of regularity of a boundary point is defined in the usual way: a point $(y, s) \in \partial G$ is said to be *regular* for L in G if, for each $f \in C(\partial G)$, the corresponding solution u of (2) satisfies

$$\lim_{(x,t) \to (y,s);(x,t) \in G} u(x, t) = f(y, s).$$

D. G. Aronson [1] has shown that every operator of the form (1) possesses a fundamental solution $g(x, t; \xi, \tau)$ and this fundamental solution satisfies

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[2] The results in this paper are taken from the author's doctoral dissertation written under the supervision of Professor D. G. Aronson and presented at the University of Minnesota, August 1970. The proofs will appear in full elsewhere.