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COMPLETENESS OF THE WAVE OPERATORS FOR SCATTERING PROBLEMS OF CLASSICAL PHYSICS[1]

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ABSTRACT. Many wave propagation phenomena of classical physics are governed by equations of the Schrödinger form $iD_t u = \Lambda u$ where

$$\Lambda = -iE(x)^{-1}\sum_{j=1}^{n} A_j D_j. \tag{1}$$

($E(x)$ and the A_j are Hermitian matrices, $E(x)$ is positive definite and the A_j are constant.) The time-evolution of such phenomena is described by a group of unitary operators $\exp(-it\Lambda)$ on the Hilbert space $\mathcal{H}$ with the energy norm

$$\|u\|_E^2 = \int_{R^n} u(x)^* E(x) u(x)\, dx. \tag{2}$$

If $E(x)$ is replaced by a constant E_0 the corresponding space and operator are denoted by H_0 and Λ_0. In this paper it is shown that the wave operators

$$W_\pm(\Lambda, \Lambda_0, J) = \operatorname*{s-lim}_{t\to\pm\infty} \exp(it\Lambda) J \exp(-it\Lambda_0) P_0^{ac} \tag{3}$$

exist and are complete if $\Lambda_0(p) = E_0^{-1}\sum_{j=1}^{n} A_j p_j$ satisfies

$$\operatorname{rank} \Lambda_0(p) = m - k \quad \text{for all } p \in R^n - \{0\}, \tag{4}$$

$E(x)$ and $D_j E(x)$ are continuous and bounded ($j = 1, 2, \cdots, n$), $E(x)$ is uniformly positive definite, $\lim_{|x|\to\infty} E(x) = E_0$ uniformly in $x/|x|$ and

$$\int_{R^n} (1 + |x|^2)^\mu |E(x) - E_0|^2 dx < \infty \quad \text{for some } \mu > n/2. \tag{5}$$

(In (3), $J: H_0 \to H$ is the identification map; $Ju = u$, and P_0^{ac} is the orthogonal projection onto the absolutely continuous subspace for Λ_0. $W_\pm$ are complete if their ranges equal H^{ac}, the absolutely continuous subspace for Λ.)

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