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CLASSIFICATION THEORY FOR HARDY CLASSES OF ANALYTIC FUNCTIONS

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I. **Introduction.** Suppose that W is an open Riemann surface. Denote by $A(W)$ and $M(W)$ the families of single-valued analytic and meromorphic functions on W, respectively. The Hardy class $H_p(W)$, for $0<p<\infty$, is the family of all $f\in A(W)$ for which $|f|^p$ admits a harmonic majorant on W. Let $AB(W)$ be the family of all bounded $f\in A(W)$. Denote by $MB^*(W)$ the family of all $f\in M(W)$ such that $\ln^+|f|$ admits a superharmonic majorant on W. Write $AB^*(W)=A(W)\cap MB^*(W)$. We shall write $W\in O_p$, O_{AB}, O_{AB^*}, O_{MB^*} whenever $H_p(W)$, $AB(W)$, $AB^*(W)$, $MB^*(W)$, respectively, reduces to the constant functions. Finally, as usual, $W\in O_G$ iff W is parabolic.

Now, as is readily verified, $AB(W)\subsetneqq H_p(W)\subsetneqq H_q(W)\subsetneqq AB^*(W)\subsetneqq MB^*(W)$ for $0<q<p<\infty$. It follows that

$$O_G \leqq O_{MB^*} \leqq O_{AB^*} \leqq \bigcap_{q>0} O_q \leqq O_p^- \leqq O_p \leqq O_p^+ \leqq \bigcup_{q<\infty} O_q \leqq O_{AB},$$

where $O_p^- = \bigcup\{O_q \mid 0<q<p\}$, $O_p^+ = \bigcap\{O_q \mid p<q<\infty\}$, $0<p<\infty$. It is known that all of these inclusions are strict in the case of arbitrary Riemann surfaces (see Heins [3, pp. 34–50] and Sario-Nakai [7, pp. 276–280]). The appropriate constructions are Myrberg type surfaces and hence of infinite genus.

If one now restricts W to be of finite genus, the situation changes. First of all, it is now known that $O_G=O_{MB^*}=O_{AB^*}$ (see Sario-Nakai [7, p. 280]). Further, Heins [3, pp. 50–51] showed next that $O_G<O_1\leqq O_{AB}$. Aside from these facts, the classification scheme for Hardy classes for Riemann surfaces of finite genus, and thus for plane domains, has remained an open question (see Heins [3, p. 50] and Rudin [6, p. 49]).

In one of our recent projects, we found a number of results on function-theoretic null-sets and classification theory for H_p classes. In this note we wish to present some of these results. Included will be a partial, though highly suggestive, answer to the open question mentioned

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