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ON THE CONVERGENCE OF MULTIPLE FOURIER SERIES

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We continue from [2].

THEOREM. *Let P be an open polygonal region in R^2, containing the origin. Set $\lambda P = \{(\lambda x, \lambda y) \mid (x, y) \in P\}$ for $\lambda > 0$. Then for*

$$f \sim \sum_{m,n=-\infty}^{\infty} a_{mn} \exp[i(mx + ny)]$$

in $L^2([0, 2\pi] \times [0, 2\pi])$, we have

$$f(x, y) = \lim_{\lambda \to \infty} \sum\nolimits_{(m,n) \in \lambda P} a_{mn} \exp[i(mx + ny)]$$

almost everywhere.

Surprisingly, this is an easy consequence of Carleson's theorem [1] on convergence of Fourier series of one variable.

PROOF. It is enough to prove the maximal inequality

$$\left\| \sup_{\lambda} \left| \sum_{(m,n) \in \lambda P} a_{mn} \exp[i(mx + ny)] \right| \right\|_2 \leqq C \|f\|_2. \tag{1}$$

Inequality (1) follows from the special case in which P is a triangle with a vertex at the origin; for any polygon breaks up into triangles, and the characteristic function of any triangle is a linear combination of characteristic functions of triangles with vertices at zero. Consequently, we can assume P has the form $P = \{(x, y) \in S \mid (x, y) \cdot t < a\}$, where S is a sector of angle $< \pi$ emanating from the origin, $t \in R^2$, and $a \in R^1$. Thus (1) is equivalent to

$$\left\| \sup_{b \in R^1} \left| \sum_{(m,n) \in S;\, (m,n) \cdot t < b} a_{mn} \exp[i(mx + ny)] \right| \right\|_2 \leqq C \|f\|_2. \tag{2}$$

Evidently, it suffices to prove (2) for rational t (with C independent of t), and to do so it is clearly enough to deal with the case $t = (p, q)$ where p and q are relatively prime integers. Finding integers r and s for which $pr - qs = 1$, we let the matrix $A = \left(\begin{smallmatrix} p & s \\ q & r \end{smallmatrix}\right) \in SL(2, Z)$ act as an automorphism of the 2-torus. Under the action of A, (2) becomes

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