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UNIFORMIZATION IN A PLAYFUL UNIVERSE

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It was shown in [1] and [3] that several questions about projective sets can be answered if one assumes the hypothesis of *projective determinacy*. We show here (in outline) that the same hypothesis settles the questions of *uniformization* and *bases* for all analytical classes.

Let $\omega = \{0, 1, 2, \cdots\}$, $R = {}^{\omega}\omega$ (the "reals"), $\mathfrak{X} = X_1 \times \cdots \times X_k$ with $X_i = \omega$ or $X_i = R$ be any product space. We study subsets of these product spaces, i.e. relations of integer and real arguments.

THEOREM 1. *Let n be odd, $n \geqq 1$, assume that every $\mathbf{\Delta}^1_{n-1}$ game is determined. Then for each Π^1_n relation $P \subseteq R \times \mathfrak{X}$, there exists a Π^1_n relation $P^* \subseteq P$ such that*

$$(\exists\alpha)P(\alpha, x) \Leftrightarrow (\exists!\alpha)P^*(\alpha, x).$$

(For $n = 1$ this is the classical Kondo-Addison Uniformization Theorem, see [8].)

There are many consequences of this result which are well known. The following computation of bases is the corollary which is foundationally most significant.

THEOREM 2. *If every projective game is determined, then every nonempty analytical set has an analytical element.*

More specifically: if n is even, $n \geqq 2$, and every $\mathbf{\Delta}^1_{n-2}$ game is determined, then every nonempty Σ^1_n subset of R contains a Δ^1_n real; if n is odd, $n \geqq 1$, and every $\mathbf{\Delta}^1_{n-1}$ game is determined, then there is a fixed real α_0 such that the singleton $\{\alpha_0\}$ is Π^1_n (so that α_0 is Δ^1_{n+1}) and every nonempty Σ^1_n subset of R contains a real recursive in α_0.

(For $n = 3$, this gives the Martin-Solovay Basis Theorem [5] with Mansfield's improvement [2]. The proofs in these two papers use only the fairly weak hypothesis that there exists a measurable cardinal, or even that for each α, $\alpha^\#$ exists. Our proof depends on the determinacy of a particular $\mathbf{\Delta}^1_2$ game and it can be verified that this game is determined if for every α, $\alpha^\#$ exists.)

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