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MANIFOLDS OF PIECEWISE LINEAR MAPS AND A RELATED NORMED LINEAR SPACE[1]

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1. **Spaces of piecewise linear maps.** Let X and Y be separable polyhedra, X compact and Y locally compact; for the moment let them be connected and of dimension >0. Denoting the separable hilbert space of square-summable sequences by l_2, a space is an l_2-*manifold* if separable, metrizable and locally homeomorphic to l_2. In [4] the author showed that the space $C(X, Y)$ of all continuous maps from X to Y with compact-open topology is an l_2-manifold. It is natural to ask whether the dense subspace $\mathrm{PL}(X, Y)$ consisting of all piecewise linear (p.l.) maps lies inside $C(X, Y)$ in some "nice" way. For example, is $\mathrm{PL}(X, Y)$ an infinite-dimensional manifold? and if so what is its model? and how are $\mathrm{PL}(X, Y)$ and $C(X, Y)$ related as manifolds?

To answer, let l_2^f be the (dense, incomplete) linear subspace of l_2 consisting of those sequences having only finitely many nonzero entries. Then we claim that $\mathrm{PL}(X, Y)$ is an l_2^f-manifold. A pair (M, N) is an (l_2, l_2^f)-*manifold pair*, if M is an l_2-manifold for which there is an open cover $\mathfrak{U}$ and open embeddings $\{f_U: U \to l_2 \mid U \in \mathfrak{U}\}$ such that for each $U \in \mathfrak{U}$, $f_U(U \cap N) = f_U(U) \cap l_2^f$. We claim that the pair $(C(X, Y), \mathrm{PL}(X, Y))$ is an (l_2, l_2^f)-manifold pair. Among other things, it follows that $\mathrm{PL}(X, Y)$ has a (metric) triangulation, and that if $\mathrm{PL}(X, Y)$ is contractible, then it is homeomorphic to l_2^f.

2. **Application: a normed linear space.** Before giving more details, we give a simple application. Consider $\mathrm{PL}(I, \boldsymbol{R})$, the space of p.l. paths in the real line, with the usual vector space structure. We have claimed that $\mathrm{PL}(I, \boldsymbol{R})$ is homeomorphic to l_2^f; but whereas the linear dimension of l_2^f is $\aleph_0$, the linear dimension of $\mathrm{PL}(I, \boldsymbol{R})$ is c. (Proof:

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[2] A special case of the theorem announced here is in the author's Ph.D. thesis written under Professor David W. Henderson at Cornell University. Supported by an I.B.M. Graduate Fellowship.