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PERTURBATIONS OF THE UNILATERAL SHIFT

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Introduction. The study of the unilateral shift on a Hilbert space has been one of the most important subjects in operator theory, for it provides many useful examples and counterexamples to all parts of Hilbert space theory (see Halmos [1]). The purpose of this note is to announce our results concerning perturbations and similarity of the unilateral shift in a slightly general setting. To be precise, let S be an isometry on a separable Hilbert space H. We were able to show that $S+P$ is similar to S for a large class $\mathfrak{R}$ of S-admissible bounded linear operators P on H. To each $P\in\mathfrak{R}$, we constructed explicitly a nonsingular bounded linear operator W on H, such that $S+P=WSW^{-1}$. In particular, the unilateral shift S on l^2 of square-summable sequences, which sends $(x_0, x_1, x_2, \cdots)$ into $(0, x_0, x_1, x_2, \cdots)$ is similar to $S+\mu P$ for all infinite matrices $P=(p_{nm})$ with $\sum |p_{nm}|<\infty$ $(n, m=0, 1, 2, \cdots \infty)$ and all sufficiently small complex parameters μ. This result becomes interesting when it is compared with that of [2], where $P=(p_{nm})$ are required to be strictly lower-triangular and $p_{n+1,n}\neq -1$ for all n, in addition to the assumption that $\sum |p_{nm}|<\infty$.

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1. S-**admissible operators.** Throughout this note, let H, H' be separable Hilbert spaces and $Y=l^2(0, \infty; H')$. We denote by $\mathfrak{B}(H, H')$ the space of all bounded linear operators on H to H'. We write $\mathfrak{B}(H)$ for $\mathfrak{B}(H, H)$. The symbol $\langle\ ,\ \rangle$ stands for the inner products in H and H'.

DEFINITION 1.1. Let S be an isometry on H and $A\in\mathfrak{B}(H, H')$. The operator A is said to be S-smooth, if there exists a constant $M<\infty$ such that

$$(1)\qquad \sum_{n=0}^{\infty}\|AS^n u\|^2 \leqq M^2\|u\|^2 \quad\text{and}\quad \sum_{n=0}^{\infty}\|AS^{*n}u\|^2 \leqq M^2\|u\|^2,$$

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