BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 4, July 1971

MORE DISTANT THAN THE ANTIPODES

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Communicated by Victor Klee, February 12, 1971

1. **Introduction.** Let X be a real normed linear space, and let $\Sigma(X)$ be its unit ball, with the boundary $\partial\Sigma(X)$. If $\dim X \geqq 2$, δ_X denotes the inner metric of $\partial\Sigma(X)$ induced by the norm (cf. [**1**, §3]). If no confusion is likely, we write Σ, $\partial\Sigma$, δ. In [**1**] we introduced and discussed parameters of X based on the metric structure of $\partial\Sigma$; among them are $D(X) = \sup\{\delta(p, q) : p, q \in \partial\Sigma\}$, the *inner diameter* of $\partial\Sigma$, and $M(X) = \sup\{\delta(-p, p) : p \in \partial\Sigma\}$, half the *perimeter* of Σ. Obviously, $M(X) \leqq D(X)$, and it was conjectured [**1**, Conjecture 9.1] that $M(X) = D(X)$ in every case, i.e., that "no pair of points of $\partial\Sigma$ is more distant in $\partial\Sigma$ than the most distant antipodes." This equality was shown to hold if $\dim X = 2$ or $\dim X = 3$ [**1**, Theorems 5.4, 5.8], if $D(X) = 4$ [**3**], if X is an L-space [**4**].

In this paper we explode this conjecture by showing that $M(X) = 2$, $D(X) = 3$ for $X = C_0((0, 1])$, the space of continuous real-valued functions on $(0, 1]$ that tend to 0 at 0, with the supremum norm. We observe that this failure of the conjecture is "as strong as possible," since $2D(X) \leqq M(X) + 4$ for every normed space X [**3**, Theorem 1]. The present result is a simple specific instance of the evaluation of $M(X)$, $D(X)$ for many spaces of continuous functions, which will be carried out in a forthcoming paper. It has appeared useful, however, to give a separate account of this very simple example. In addition, Lemma 1 is required for the general theory. The conjecture remains unresolved, and interesting, for spaces of finite dimension greater than three.

We shall use the terminology, notations, and elementary results of §§1–3 of [**1**]. In particular, a *subspace* of X is a linear manifold in X, not necessarily closed, provided with the norm of X. If Y is a subspace of X, we obviously have

$$\delta_Y(p, q) \geqq \delta(p, q), \qquad p, q \in \partial\Sigma(Y). \tag{1}$$

Instead of dealing with the space $C_0((0, 1])$, we prefer, for technical reasons, to consider the space $C_\pi([-1, 1])$ of odd continuous real-valued functions on $[-1, 1]$ with the supremum norm. The two

AMS 1970 *subject classifications*. Primary 52A40; Secondary 46E15.
Key words and phrases. Inner metric on unit sphere, inner diameter, perimeter.
[1] This work was supported in part by NSF Grant GP-19126.