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K-THEORY OF A SPACE WITH COEFFICIENTS IN A (DISCRETE) RING

BY DAVID L. RECTOR[1]

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In [2], [3], S. Gersten has introduced higher K-groups of a ring which satisfy properties analogous to those of a generalized homology theory in a suitably defined homotopy category of rings [1]. In this announcement we use Gersten's K-groups to define for a ring R a generalized cohomology theory $K_R^*(\ )$, analogous to the Atiyah-Hirzebruch K-theory, on the category of finite simplicial sets so that $K_R^*(pt)=K_\Sigma^*R$, where K_Σ^*R are Gersten's stable K-groups of the ring R. If R is suitably restricted, in particular if it is commutative and regular, the theory $K_R^*(\ )$ will have products and Adams operations. One may also define, using the continuous theory in [6], a K-theory $K_\Lambda^*(\ )$ with coefficients in a Banach ring Λ. This theory coincides with the Atiyah-Hirzebruch theory for $\Lambda=\boldsymbol{R}$, $\boldsymbol{C}$, or $\boldsymbol{H}$. We give here an outline of proofs. A full account will appear elsewhere.

1. **Definition of the theory.** We recall the definition of Gersten's theory as given in [5]. Let R be a ring (without unit). The functor $R\mapsto R[t]$ together with the natural transformations $R[t]\to R$ via "$t\to 1$", and $R[t]\to R[t, t']$ via $t\to tt'$ define a cotriple in the category of rings. If ER is the ideal $R[t]t$, then the restriction of those maps makes the functor $R\mapsto ER$ a cotriple. Associated to these cotriples are canonical simplicial rings $R[T]$ and $\overline{E}R$ with

$$R[T]_n = R[t_0, \cdots, t_n], \qquad \overline{E}R_n = E^{n+1}R.$$

Let QR be the simplicial ring

$$QR = R[T]/\overline{E}R.$$

One has

$$K^{-i-1}R = \pi_i \operatorname{Gl} QR$$

where Gl denotes the general linear group functor. This K-theory of rings is stabilized as follows [3]. Let ΓR be the kernel of $R[t, t^{-1}]\to R$. Then there is a natural homomorphism

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