

INFINITE RESISTIVE NETWORKS

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An *infinite resistive network* N consists of a connected, locally finite, oriented, infinite graph with branches $B_1, B_2, \dots$. To each branch B_i is associated a resistance $r_i \geq 0$. We are also given a voltage source, i.e., a finite 1-cochain E' , and a current source, i.e., a finite 0-chain i satisfying $\partial_0 i = 0$.

For each (real) 1-chain $C = \sum a_i B_i$, define $\|C\|$ by $\|C\|^2 = \sum a_i^2 r_i$.

THEOREM 1. *There exists a unique 1-chain I such that:*

- (i) (Kirchhoff's current law). $\partial I + i = 0$.
- (ii) (Kirchhoff's voltage law). For each finite cycle Z ,

$$\langle E', Z \rangle = \langle R(I), Z \rangle,$$

where if $I = \sum a_j B_j$, then $R(I)$ denotes the 1-cochain $R(I) = \sum a_j B'_j$. Of course $\langle B'_j, B_i \rangle = \delta_{ij}$.

- (iii) (Finite power). I is square summable, i.e., $\|I\| < \infty$.

(iv) There is a sequence $\{C_j\}$ of finite 1-chains such that $\partial C_j + i = 0$ and $\|C_j - I\| \rightarrow 0$.

THEOREM 2. *Let N_j be any sequence of subnetworks such that $N_1 \subset N_2 \subset \dots$ and $\bigcup N_j = N$. Suppose N_1 is large enough to support the voltage source E' and the current source i . Let I_j be the unique current on N_j given by Theorem 1. Then $\|I_j - I\| \rightarrow 0$, where I is the unique current on N .*

The proofs of these results, corollaries, and a full discussion will appear shortly in the IEEE Trans. Circuit Theory.

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