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JORDAN TRIPLE SYSTEMS, R-SPACES, AND BOUNDED SYMMETRIC DOMAINS

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ABSTRACT. In this note, we establish a one-to-one correspondence between compact Jordan triple systems (see below for the definition) and symmetric R-spaces (i.e., symmetric spaces which are quotients of semisimple Lie groups by parabolic subgroups, see [7]). We obtain a simple geometric characterization of symmetric R-spaces among compact symmetric spaces. The noncompact dual of a symmetric R-space may be realized as a bounded domain D in a real vector space. There is a one-to-one correspondence between boundary components of D and idempotents of the corresponding Jordan triple system. Using this, we generalize the results of Wolf-Koranyi [8] to the real case. In particular, the image of D under a generalized Cayley transformation is the real equivalent of a Siegel domain of type III.

1. **Jordan triple systems.** A Jordan triple system (=JTS) (see [2], [3]) is a vector space V together with a trilinear map $V\times V\times V \to V$, $(x, y, z)\mapsto \{xyz\}$, satisfying the following identities:

$$\{xyz\} = \{zyx\}, \tag{1}$$

$$\{uv\{xyz\}\} = \{\{uvx\}yz\} - \{x\{vuy\}z\} + \{xy\{uvz\}\}. \tag{2}$$

For $x, y\in V$ we define the linear map $L(x, y)$ of V by $L(x, y)(z) = \{xyz\}$. A finite-dimensional real JTS is called *compact* if the quadratic form $x\mapsto \operatorname{trace} L(x, x)$ is positive definite. From now on, V will denote a compact JTS. Then V becomes a Euclidean vector space with the scalar product $(x, y) = \operatorname{trace} L(x, y)$. By (2), the vector space $\mathfrak{H}$ spanned by $\{L(x, y):x, y\in V\}$ is a Lie algebra of linear transformations of V, closed under taking transposes with respect to $(\,,\,)$. Thus the contragredient $\mathfrak{H}$-module V' of V may be identified with V as a vector space, and $X\cdot v' = -{}^tX(v')$, for $X\in\mathfrak{H}$ and $v'\in V'$.

THEOREM 1 (KOECHER). (a) *$\mathfrak{L} = V+\mathfrak{H}+V'$ becomes a semisimple Lie algebra with the definitions*

$$[X, Y] = XY - YX, \qquad [X, v] = -[v, X] = X\cdot v,$$

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