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DUALITY AND VON NEUMANN ALGEBRAS[1]

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The aim of this paper is to announce the results of the author's lecture given in Tulane University for the Fall of 1970 under the same title. Since the Pontryagin duality theorem was shown, a series of duality theorems for nonabelian groups has been discovered, Tannaka duality theorem [**14**], Stinespring duality theorem [**10**], Eymard-Saito duality theorem [**5**], [**8**] and Tatsuuma duality theorem [**15**]. Motivated by the Stinespring duality theorem, Kac [**7**] introduced the notion of "ring-groups" in order to clarify the duality principle for *unimodular* locally compact groups. Sharpening and generalizing Kac's postulate for the "ring-group," the author [**11**] gave a characterization of the group algebra of a *general* locally compact group as an involutive abelian Hopf-von Neumann algebra with left invariant measure.

Let G be a locally compact group with left Haar measure ds. Let $\mathfrak{H}$ denote the Hilbert space $L^2(G, ds)$. Define a unitary operator W on $\mathfrak{H}\otimes\mathfrak{H}$ by $(Wf)(s, t)=f(s, st), f\in\mathfrak{H}\otimes\mathfrak{H}, s, t\in G$. Let $\mathcal{A}(G)$ be the von Neumann algebra on $\mathfrak{H}$ consisting of all multiplication operators $\rho(f)$ by $f\in L^\infty(G)$. The algebras $\mathcal{A}(G)$ and $L^\infty(G)$ will be identified. Let $\mathfrak{M}(G)$ denote the von Neumann algebra on $\mathfrak{H}$ generated by left regular representation λ of G. The fundamental facts of all duality arguments for groups are the following: the map $\delta_G: x\mapsto W(x\otimes 1)W^*$ is an isomorphism of $\mathcal{A}(G)$ into $\mathcal{A}(G)\,\overline{\otimes}\,\mathcal{A}(G)$ such that $(\delta_G\otimes i)\circ\delta_G=(i\otimes\delta_G)\circ\delta_G$; the map $\gamma_G: x\mapsto W^*(x\otimes 1)W$ is an isomorphism of $\mathfrak{M}(G)$ into $\mathfrak{M}(G)\,\overline{\otimes}\,\mathfrak{M}(G)$ such that $(\gamma_G\otimes i)\circ\gamma_G=(i\otimes\gamma_G)\circ\gamma_G$ and $\sigma\circ\gamma_G=\gamma_G$ where σ denotes the automorphism of $\mathcal{B}(\mathfrak{H})\,\overline{\otimes}\,\mathcal{B}(\mathfrak{H})$ defined by $\sigma(x\otimes y)=y\otimes x$. According to these facts, the preduals $\mathcal{A}_*(G)$ and $\mathfrak{M}_*(G)$ of $\mathcal{A}(G)$ and $\mathfrak{M}(G)$ turn out to be Banach algebras by the following multiplications: $\langle x, \psi*\varphi\rangle=\langle\delta(x), \psi\otimes\varphi\rangle$, $x\in\mathcal{A}(G)$, $\psi, \varphi\in\mathcal{A}_*(G)$ and $\langle x, \varphi*\psi\rangle=\langle\gamma(x), \varphi\otimes\psi\rangle$, $x\in\mathfrak{M}(G)$, $\varphi, \psi\in\mathfrak{M}_*(G)$. The Banach algebra $\mathcal{A}_*(G)$ is nothing but the usual group algebra $L^1(G)$ and the duality theorems mention that the Banach algebra $\mathfrak{M}_*(G)$ is semisimple and the spectrum space

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