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A NONSTANDARD REPRESENTATION OF MEASURABLE SPACES AND L_∞

BY PETER A. LOEB[1]

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The results given in this note were obtained by applying to measure theory the methods of nonstandard analysis developed by Abraham Robinson [5]. Amplifications of these results with proofs will be published elsewhere.[2] It is shown here that there are linear mappings from an arbitrary, real L_∞ space and its dual L_∞^* into Euclidean ω-space E^ω, where ω is an infinite integer. Finite valued, finitely additive measures on the underlying measurable space are also mapped onto elements of E^ω, and integrals are infinitesimally close to the corresponding inner products in E^ω. Yosida and Hewitt's representation of L_∞^* [6] is an immediate consequence of these results.

In general, we use Robinson's notation [5]. If we have an enlargement of a structure that contains the set R of real numbers, then $^*R$ denotes the set of nonstandard real numbers and $^*N$, the set of nonstandard natural numbers. A set S is called *finite if there is an internal bijection from an initial segment of $^*N$ onto S; a *finite set has all of the "formal" properties of a finite set. Given b and c in $^*R$, we write $b \simeq c$ if $b-c$ is in the monad of 0; when b is finite, we write $^\circ b$ for the unique, standard real number in the monad of b.

1. The partition P and bounded measurable functions. Let X be an infinite set and $\mathfrak{M}$ an infinite σ-algebra of subsets of X. Fix an enlargement of a structure that contains X, $\mathfrak{M}$, and the extended real numbers. There is a *finite, $^*\mathfrak{M}$-measurable partition P of $^*X$ such that P is finer than any finite $\mathfrak{M}$-measurable partition of X. That is, $P \subset {}^*\mathfrak{M}$ has the following properties:

(i) There is an infinite integer $\omega_P \in {}^*N$ and an internal bijection from $I = \{i \in {}^*N: 1 \leqq i \leqq \omega_P\}$ onto P. Thus we may write $P = \{A_i: i \in I\}$.

(ii) If i and j are in I and $i \neq j$, then $A_i \neq \emptyset$ and $A_i \cap A_j = \emptyset$.

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[2] These results were announced at the 1970 Oberwolfach conference on nonstandard analysis.