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SOME NONLINEAR STOCHASTIC GROWTH MODELS

BY HARRY KESTEN

ABSTRACT. Stochastic growth models which generalize Galton-Watson branching processes are discussed. The models have an interpretation in population dynamics and economics. The individuals or particles *do* interact, e.g. if the individuals represent members of a population, allowance is made for sexual reproduction, so that pairs of individuals are needed to produce offspring. The typical form of the results is that with probability one, either the population remains bounded in size, or it grows at an exponential rate and its composition converges to a fixed point of a suitable transformation.

1. **Introduction.** The problem to which we want to address ourselves is "What is the rate of the population explosion?" In order to deal with this problem mathematically one needs a more or less explicit model for population growth, and we shall consider some probabilistic ones. Stochastic models describing the growth and/or extinction of various populations abound and we only list some of the (historically) more important references and some books or survey articles in which the reader can find further references: Fisher (1930) (Chapter IV has one of the earliest biological applications of branching processes), Feller (1939) (first mathematical treatment of fairly general birth and death processes), Feller (1950) (cf. especially his comments in §§1, 6 and 7 on the difficulty of treating size *and* composition of a population with several interacting types), Bartlett (1949) (use of generating functions to handle birth and death processes), Bartlett (1960), Kendall (1949) (cf. especially §2(ix), where the problem of two sexes is discussed), Goodman (1953) and (1968) (considers two sexes with age specific birth and death rates, but has no interactions between the sexes in his stochastic models), Bailey (1957) (models for epidemics), Bharucha-Reid (1960) (Chapter 4 summarizes several models and gives many references), Moran (1962) (especially

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