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MULTIPLICATIVE OPERATOR FUNCTIONALS OF A MARKOV PROCESS

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1. **Introduction.** Let $X=(x(t),\ \zeta,\ \mathfrak{M}_t,\ P_x)$ be a right continuous Markov process on a state space $(E,\ \mathcal{B})$.[2] Let L be a fixed Banach space. A *multiplicative operator functional* (MOF) of $(X,\ L)$ is a mapping $(t,\ \omega)\rightarrow M(t,\ \omega)$ of $[0,\ \infty)\times\Omega$ to bounded operators on L which possesses the following properties:

(1a) $\omega\rightarrow M(t,\ \omega)f$ is $\mathfrak{M}_t$ measurable for each $t\geqq 0, f\in L$.

(1b) $t\rightarrow M(t,\ \omega)f$ is right continuous a.s. for each $f\in L$.

(1c) $M(t+s,\ \omega)f=M(t,\ \omega)M(s,\ \theta_t\omega)f$ a.s. for each $s,\ t\geqq 0, f\in L$.

(1d) $M(0,\ \omega)$ is the identity operator on L.

If M is a multiplicative operator functional of $(X,\ L)$ the *expectation semigroup* is defined on the direct sum Banach space $\tilde{L}=\bigoplus_E L$ by the equation

$$(\tilde{T}(t)\tilde{f})_x = E_x[M(t,\ \omega)f_{x(t,w)}]. \tag{1.1}$$

The MOF concept has appeared in several places recently. We were led to the idea by the work of Griego and Hersh [4], [5] who constructed examples of an MOF when X is a Markov chain with a finite number of states and L is arbitrary. Here $M(t,\ \omega)$ is a finite random product of semigroups. Earlier Babbitt [1] had studied the case $X=$Wiener process on R^n, $L=$finite-dimensional vector space. In this case $M(t)$ is a solution to a system of Itô stochastic differential equations. If X is a Poisson process and L is a Banach space of continuous functions on R^1, we can specialize the MOF concept to represent the semigroups studied by Çinlar and Pinsky in a problem in storage theory. In this case the infinitesimal operator of the associated semigroup is an integro-differential operator. Further applications will be discussed in another publication.

2. **Main results.** Here we will give the notations and state the main results. Proofs will not be given. Detailed proofs will appear elsewhere.

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[2] See Dynkin [3] for the definition and notations for Markov processes.