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APPLICATIONS OF THE SEMISIMPLE SPLITTING

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Let S be a solvable simply connected analytic group. A closed subgroup C of S will be called uniform in S if the quotient space S/C is compact. We will make the assumption throughout the rest of the paper that a closed uniform subgroup C of S has the property that it contains no normal analytic subgroup of S. Let C be a closed uniform subgroup of S. In [4], G. D. Mostow proved that the intersection of C with the nil-radical of S must be uniform in the nil-radical of S. L. Auslander in [6], [7] exploited this fact to relate the structure of C to the structure of S. The object of this paper is to show how the semisimple splitting of S, introduced in [1] and studied further in [2] and [3], provides a convenient language with which to deal with the main theorems of [4], [6] and [7].

Preliminaries. We shall need the following ideas from the theory of algebraic groups. Let X, Y, and Z be subgroups of $\mathrm{Gl}(n, R)$, the group of all n by n real matrices. We denote the algebraic hull of X by $\mathfrak{A}(X)$, and the group of commutators of X and Y by (X, Y). Then

(a) $\mathfrak{A}((X, Y)) = \mathfrak{A}((\mathfrak{A}(X), \mathfrak{A}(Y)))$.

(b) If X is a solvable algebraic group then we can write

$$X = U \cdot T \quad \text{(semidirect product)}$$

where U is the collection of all unipotent matrices in X and T is a maximal completely reducible subgroup of X. Moreover if $T^\#$ is another maximal completely reducible subgroup of X then there is an x in U such that $xTx^{-1} = T^\#$.

We will require also the following facts from the theory of nilpotent Lie groups. By a real nilpotent group N we mean a simply connected nilpotent analytic group. We denote the Lie algebra of N by $L(N)$. The exponential map defines a homeomorphism of $L(N)$ onto N. By a lattice of a vector space V we mean the collection of all integer combinations of a basis of V. A discrete uniform subgroup C of N is called

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