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THE HILBERT BALL AND BI-BALL ARE HOLOMORPHICALLY INEQUIVALENT[1]

BY STEPHEN J. GREENFIELD AND NOLAN R. WALLACH

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1. **Introduction.** In this note we prove that if B is the unit ball of a complex Hilbert space then B and $B\times B$ are holomorphically inequivalent. This answers a question of Burghelea. We also announce some results on the automorphism groups of bounded domains in a Hilbert space.

2. **The ball and the bi-ball.** Let H be a complex Hilbert space. Let $B=\{z\in H|\ \|z\|<1\}$. Here $\|\cdots\|$ is the Hilbert norm, and we denote by $\langle\ ,\ \rangle$ the inner product on H.

THEOREM 2.1. *B and $B\times B$ are holomorphically inequivalent. (That is, there is no diffeomorphism $f:B\to B\times B$ so that $df(z)$ is complex linear for each $z\in B$.)*

PROOF. Suppose that $f:B\to B\times B$ is a holomorphic equivalence. We derive a contradiction. We first assert that we may assume that $f(0)=(0,0)$. Indeed, suppose that $f(z)=(0,0)$, $z\in B$. Define for $w\in B$, $w=w_1+\lambda z$, $\langle w_1, z\rangle=0$,

$$h(w)=\frac{(1-\|z\|^2)^{1/2}w_1+(\lambda+1)z}{\lambda\|z\|^2+1}.$$

It is not hard to check that $h:B\to B$ is a holomorphic self-equivalence and $h(0)=z$. Replace f by $f\circ h$. Then $f(0)=(0,0)$.

Let $z\in B$. Then

$$f(\lambda z)=\sum_{k=1}^{\infty}(\lambda^k/k!)\,d^kf(0)z^k\quad\text{for }|\lambda|\leqq 1 \tag{1}$$

and the convergence is uniform on $|\lambda|\leqq 1$. (Here $d^kf(0)$ is the kth derivative of f, z^k is the k-tuple $(z,\cdots,z)$.)

Set

$$G(z)=\frac{1}{2\pi}\int_0^{2\pi}e^{-i\theta}f(e^{i\theta}z)\,d\theta.$$

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