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SETS OF INTERPOLATION FOR MULTIPLIERS

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Let T denote the circle and I a closed ideal of $L^1(T)$ under convolution. Let $\mathfrak{F}I$ denote the set of sequences of complex numbers which are Fourier transforms of elements of I.

$$\mathfrak{F}I = \{(\xi_n): \exists f \in I, \ \hat{f}(n) = \xi_n\}.$$

A subset E of the integers is called a set of *interpolation* for the multipliers of $\mathfrak{F}I$ $(=M(\mathfrak{F}I))$ if every bounded complex sequence defined on E is the restriction to E of a multiplier of $\mathfrak{F}I$. E is called a *Sidon* set if every bounded complex sequence on E is the restriction to E of the Fourier transform of some measure on T. Answering a question of Y. Meyer we show here that every set of interpolation $E \subseteq Z^+$ for $M(\mathfrak{F}H^1(T))$ is a Sidon set.

Let $A(T)$ denote the Banach space of all analytic continuous functions on T equipped with the supremum norm. Let $\beta = H^1(T) \hat{\otimes} C(T)$ be the Banach space of all elements of $A(T)$ which can be expressed in the form $\sum_1^\infty f_k * g_k$ where $f_k \in H^1(T)$, $g_k \in C(T)$ and such that $\sum_1^\infty \|f_k\|_1 \|g_k\|_\infty < \infty$. The norm $\|\cdot\|_\beta$ in β is the infimum over all such representations. Meyer [1] has shown that the dual of β is precisely $M(\mathfrak{F}H^1(T))$.

THEOREM 1. *β is isometrically isomorphic to $A(T)$.*

PROOF. It is clear that the natural embedding of β in $A(T)$ is norm decreasing. Let $P(\theta) = \sum_1^\nu a_k \exp[in_k\theta]$ be an arbitrary analytic trigonometric polynomial and write $e^{iM\theta}P(\theta)$ as

$$\sum_{n=-N}^{N} \left(1 - \frac{|n|}{N}\right) \exp[i(n+N)\theta] * \sum_{k=1}^{\nu} b_k \exp[i(n_k + M)\theta]$$

where $b_k = a_k\{1 - |n_k + M - N|/N\}^{-1}$. Choose $M = N - [N^{1/2}]$ and N larger than n_ν. It is clear that as $N \to \infty$, $b_k \to a_k$ for each k. Since the polynomial on the left-hand side is just a translate of the usual Fejer kernel, it has L^1 norm equal to 1. By the choice of M, the sup norm of the polynomial on the right-hand side tends to $\|P(\theta)\|_\infty$ as $N \to \infty$. Hence

$$\|\exp[iM\theta]P(\theta)\|_\beta < \|P(\theta)\|_\infty + \epsilon$$

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