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THE STRUCTURE OF ω-REGULAR SEMIGROUPS

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1. Finding the complete structure of regular semigroups of a certain class has succeeded only when sufficiently strong conditions on idempotents and/or ideals have been imposed. On the one hand, there is the theorem of Rees [7], giving the structure of completely 0-simple semigroups, and its successive generalizations to primitive regular semigroups [2], and $\mathfrak{J}$- and $\mathfrak{J}_1$-regular semigroups [4]. On the other hand, with very different restrictions, Reilly [8] has determined the structure of bisimple ω-semigroups, Kočin [1] of inverse simple ω-semigroups, Munn [5] of inverse ω-semigroups.

An ω-chain with zero is a poset $\{e_i \mid i \geqq 0\} \cup 0$ with $e_i > e_j$ if $i < j$, and $0 < e_i$ for all i, j. We call a regular semigroup S *ω-regular* if S has a zero and the poset of its idempotents is an orthogonal sum [2] of ω-chains with zero. We announce here the complete determination of the structure of such semigroups, including various special cases thereof, and briefly mention their isomorphisms.

2. An ω-regular semigroup can be uniquely written as an orthogonal sum of ω-regular prime (i.e., with 0 a prime ideal) semigroups. This reduces the problems of structure and isomorphism to ω-regular prime semigroups. We distinguish three cases: (i) 0-simple, (ii) prime with a proper 0-minimal ideal, (iii) prime without a 0-minimal ideal. Case (i) is the most difficult (and interesting) and includes a variety of special cases some of which reduce to those constructed by Reilly [8], Kočin [1], and Munn [5], [6].

3. Let A be a nonempty set, d be a positive integer, V be a semigroup which is a chain of d groups $G_0 > G_1 > \cdots > G_{d-1}$, and σ be a homomorphism of V into G_0. Let $w: A \rightarrow \{0, 1, \cdots, d-1\}$ be any function, denoted by $w: \alpha \rightarrow w_\alpha$. For $\alpha \in A$, $0 \leqq i, j < d$, define $\langle \alpha, i \rangle$ by

$$\langle \alpha, i \rangle \equiv w_\alpha + i \pmod{d}, \qquad 0 \leqq \langle \alpha, i \rangle < d,$$

and define $[i, \alpha, j]$ to satisfy

$$[i, \alpha, j]d = (i - j) - (\langle \alpha, i \rangle - \langle \alpha, j \rangle).$$

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