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BOOK REVIEWS

Maximum principles in differential equations by M. H. Protter and H. F. Weinberger. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1967. x+261 pp. $8.00.

This book is devoted to the study of maximum principles in partial differential equations. It contains a wealth of material much of which is presented for the first time in a book form. An attractive feature of the book is that it is completely elementary and thus accessible to a wide audience of readers.

The book has four chapters. Chapter I deals with the one dimensional maximum principle. The discussion of this very simple model of a maximum principle forms a good introduction to the general theory. Various applications of the principle are given to show that it is a very useful tool even in the study of ordinary differential equations. As an example, it is shown that many oscillation and comparison results in the Sturm-Liouville theory could be deduced most easily by a maximum principle argument.

The proper discussion of maximum principles in partial differential equations begins in Chapter II. This chapter, which is the backbone of the book, is devoted to elliptic equations. The material covered in this chapter includes the E. Hopf maximum principle and its generalizations; the Phragmèn-Lindelöf principle for solutions of elliptic equations; Serrin's version of the Harnack inequality for solutions of general elliptic equations in two variables (this is probably the most difficult result discussed in the book); various versions of the Hadamard three circles theorems for solutions of elliptic equations; applications of the maximum principle to nonlinear equations and to problems of fluid flow.

Chapter III is devoted to parabolic equations. The plan of this chapter parallels that of Chapter II. The topics discussed include the L. Nirenberg strong maximum principle; a three curves theorem with an interesting application to the Tikhonov uniqueness theorem; a Phragmèn-Lindelöf principle for parabolic equations with applications to uniqueness results; nonlinear operators; a maximum principle for certain parabolic systems.

The fourth and the last chapter is devoted to hyperbolic equations. The results in this chapter are somewhat special since a maximum principle in the proper sense does not hold for solutions of hyperbolic equations. Nevertheless, solutions of certain hyperbolic equations