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CURVATURE AND DIFFERENTIABLE STRUCTURE ON SPHERES[1]

BY ERNST A. RUH

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1. **Introduction.** The purpose of this note is to outline a proof of the following result: A simply connected, complete, riemannian manifold whose curvature tensor R is sufficiently close to the curvature tensor R_0 of the standard sphere S of the same dimension is diffeomorphic to S. Traditionally, the proximity of R and R_0 has been measured in terms of the sectional curvature as follows: A riemannian manifold is called δ-pinched if the sectional curvature K satisfies the condition $\delta < K \leqq 1$. Using this concept, Gromoll [4] and Calabi proved the following diffeomorphism theorem: There exists a sequence δ_n with $\lim \delta_n = 1$ as n increases such that a δ_n-pinched simply connected riemannian manifold M of dimension n is diffeomorphic to the sphere S^n.

In order to express the main condition of the diffeomorphism theorem independently of dimension, we introduce a different measurement for the proximity of the curvature tensors R and R_0 of the manifolds M and S^n respectively. To formulate this condition we think of the riemannian curvature tensor as a selfadjoint, linear map $R: V \wedge V \rightarrow V \wedge V$, where $V \wedge V$ denotes the exterior product of the tangent space with itself. A riemannian manifold is called *strongly δ-pinched*, if the eigenvalues λ of the above linear map at every point of M satisfy the condition $\delta < \lambda \leqq 1$.

2. **Statement of result.** In previous studies the pinching constant depended on the dimension of the manifold. However, the introduction of strong δ-pinching has the following advantage: The constant δ in the theorem below is independent of the dimension of the manifold.

THEOREM. *There exists a constant $\delta \neq 1$ such that a complete, simply connected, strongly δ-pinched riemannian manifold is diffeomorphic to the standard sphere of the same dimension.*

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