THE HOMOLOGY OF THE CLASSICAL GROUPS
OVER THE DYER-LASHOF ALGEBRA[1,2]

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1. Introduction. By Bott periodicity the classical groups, their classifying spaces, their homogeneous spaces, Im J and B Im J are infinite loop spaces, and hence the Dyer-Lashof operations act on their mod p homology for p a prime number. We will list the basic properties of the Dyer-Lashof operations in §2. These operations have been calculated in the homology of all these spaces, and the dual operations have also been computed. As applications one can calculate the indecomposable elements of the homology of these spaces over the Dyer-Lashof algebra R and the AR-Hopf algebra maps between any two classical groups or any two of their classifying spaces. In this paper we will summarize our results for BU, BU (the connected and infinite component versions of the classifying space of the infinite unitary group), O, SO, B Im J and Im J.

These results have been applied to the study of $H_*(F)$ and $H_*(BF)$ by J. P. May [9] and I. Madsen [7]. They are also useful in cobordism theory (see for example T. tom Dieck [5, p. 396]).

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NOTATION. All elements of a graded object will be indexed by their degree with the exception of the Chern and Wu classes. All homology and cohomology will have Z_p coefficients for p a prime number. When a result differs for p an odd prime and $p = 2$ then the result for $p = 2$ will be placed in square brackets.

2. The Dyer-Lashof operations. The homology of an infinite loop space B has natural homomorphisms $Q^i : H_*(B) \to H_*(B)$ for $i \geq 0$ of degree $2i(p-1)$ [of degree i] which have been studied by S. Araki and