BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 1, January 1971

POINTWISE BOUNDED APPROXIMATION AND HYPODIRICHLET ALGEBRAS[1]

BY T. W. GAMELIN[2] AND JOHN GARNETT

Communicated by F. W. Gehring, July 2, 1970

1. Introduction. Let U be an open subset of the Riemann sphere S^2. The algebra of bounded analytic functions on U is denoted by $H^\infty(U)$ and the algebra of continuous functions on $\bar{U}$ which are analytic on U is denoted by $A(U)$. We are interested in the following two questions:

(1) When does Re $A(U)$ have finite defect in $C_R(\partial U)$? That is, when does the uniform closure of the real parts of functions in $A(U)$ have finite codimension in the space of continuous real-valued functions on the boundary ∂U of U?

(2) When is $A(U)$ pointwise boundedly dense in $H^\infty(U)$? That is, when can every function in $H^\infty(U)$ be approximated pointwise on U by a bounded sequence in $A(U)$?

These two problems are related by the following theorem, which extends a result of A. M. Davie [2].

THEOREM 1. *Suppose ∂U has no isolated points. Then* Re $A(U)$ *has finite defect in $C_R(\partial U)$ if and only if $S^2\backslash U$ has a finite number of components and $A(U)$ is pointwise boundedly dense in $H^\infty(U)$.*

In this announcement, we wish to elaborate on this result, and to state answers to questions (1) and (2) in terms of analytic capacity.

2. A theorem on uniform approximation. By K we will always denote a compact subset of the complex plane. The algebra $R(K)$ is the uniform closure on K of the rational functions with poles off K. The algebra $A(K)$ consists of the continuous functions on K which are analytic on the interior K° of K. With this notation, $A(K)$ consists of the functions in $A(K^\circ)$, extended in all possible continuous ways to K. Consequently we can deduce from each theorem about $A(U)$ a corresponding theorem about $A(K)$, by setting $U=K^\circ$. In

AMS 1970 *subject classifications*. Primary 41A20, 41A30, 46J10.

Key words and phrases. Pointwise bounded approximation, uniform approximation, rational functions, analytic capacity, dirichlet algebras.

[1] The preparation of this announcement was sponsored in part by NSF Grant GP-11475.

[2] Fellow of the Alfred P. Sloan Foundation.