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CONVERGENCE, SUMMABILITY, AND UNIQUENESS OF MULTIPLE TRIGONOMETRIC SERIES

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1. Relationships between methods of convergences and the growth of coefficients. It was shown by Paul J. Cohen [1] that if a multiple trigonometric series converges regularly at almost every point of the k-torus $T^k=[-\pi, \pi]\times\cdots\times[-\pi, \pi]$, then its coefficients $a_n=a_{n_1,\dots,n_k}$ cannot exhibit exponential growth. A particular form of regular convergence is square convergence. Consideration of double series of the form

$$\sum_{n=1}^{\infty} \phi(n)(1-\cos x)^n e^{iny}$$

shows that Cohen's seemingly gross estimates cannot be improved. For by a suitable choice of the $\phi(n)$ the series may be made square convergent almost everywhere while having coefficients which grow faster than any given sequence whose growth is less than exponential.

THEOREM 1. *If a multiple trigonometric series converges unrestrictedly rectangularly on a set, then the coefficients are necessarily bounded; furthermore,* $a_n=a_{n_1,\dots,n_k}\to 0$ *as* $\min\{|n_1|, \cdots, |n_k|\}=\|n\|\to\infty$.

Again this theorem is best possible. The proof is by induction and makes use of

LEMMA 1. *If a polynomial* $P(e^{ix})$ *of degree* n *is bounded for all* $x\in E\subset[0, 2\pi)$ *by a bound* B, *where* $|E|$ *=Lebesgue measure of* $E=\delta>0$, *then there is a number* $c=c(\delta, n)$ *such that* $|P(e^{ix})|\leq c$ *for every* x.

The lemma is an easy consequence of the Lagrange interpolation formula and a lemma of Paul Cohen's [1, p. 41]. Another consequence of Lemma 1 is

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