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THE REDUCIBILITY OF THOM COMPLEXES AND SURGERY ON MAPS OF DEGREE d

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1. **Statement of results.** In this paper we announce a substantial extension of the obstruction theory developed in [1]. Detailed proofs and further properties will appear elsewhere. Basically what we have accomplished is to put the theory in [1] in a more natural setting and extend it to the metastable range. We also apply it to the question of when the Thom complex $T(\xi)$ of a vector bundle ξ over a manifold is reducible and hence get an obstruction theory for imbedding 1-connected manifolds in the metastable range.

All manifolds will be oriented, compact, and differentiable. Bundles are also assumed to be oriented. Let θ^q denote the trivial q-plane bundle over a space and set $\pi_{r,t}=\pi_r S^{r-t}$.

THEOREM 1 (THE REDUCIBILITY THEOREM). *Let ξ^k be a k-plane bundle over a closed connected manifold M^n and assume $2k\geqq n+3$. Then*

(a) *The Thom complex $T(\xi\oplus\theta^1)$ is reducible whenever a sequence of obstructions $o_i(\xi)\in H^i(M;\ \pi_{n+k,i-1})$, $1<i\leqq n$, vanishes. In the stable range, $k\geqq n+3$, the converse is also true, namely, the reducibility of $T(\xi\oplus\theta^1)$ implies the vanishing of the obstructions $o_i(\xi)$.*

(b) *Let $s_*:H^i(M;\ \pi_{n+k,i-1})\to H^i(M;\ \pi_{n+k+1,i-1})$ be the homomorphism induced by the suspension homomorphism $\pi_{n+k,i-1}\to\pi_{n+k+1,i-1}$. Then $s_*(o_i(\xi))=o_i(\xi\oplus\theta^1)$.*

Next, let $G_{r,t}$ denote the kernel of the suspension homomorphism $\pi_{r,t}\to\pi_t^s=\lim_{q\to\infty}\pi_{r+q,t}$ and let $G^q_{r,t}$, for $q\leqq r$, be the subgroup of those elements in $G_{r,t}$ which lie in the image of the suspension homomorphism $\pi_{q,t}\to\pi_{r,t}$. Then $G^q_{r,t}\subseteq G_{r,t}\subseteq\pi_{r,t}$.

Combining Theorem 1 with [6], we get:

THEOREM 2 (THE IMBEDDING THEOREM). *Let M^n be a 1-connected closed manifold that immerses in S^{n+k} with normal bundle ξ^k, where $n\geqq 5$*

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